Currículo para o Século XXI 2017

Conferência Internacional

Pensar a Matemática

How can Mathematics Education prepare students for their future?

Koeno Gravemeijer

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Overview

- Robotization, computerization, globalization
- Mathematics in the 21e-century society
- General Goals: 21st Century Skills
  - problem-centered, interactive math ed.
- Mathematics Education Goals:
  - 1. Context (workplace)
  - 2. Competencies
  - 3. Content
  - mathematics for everyday life

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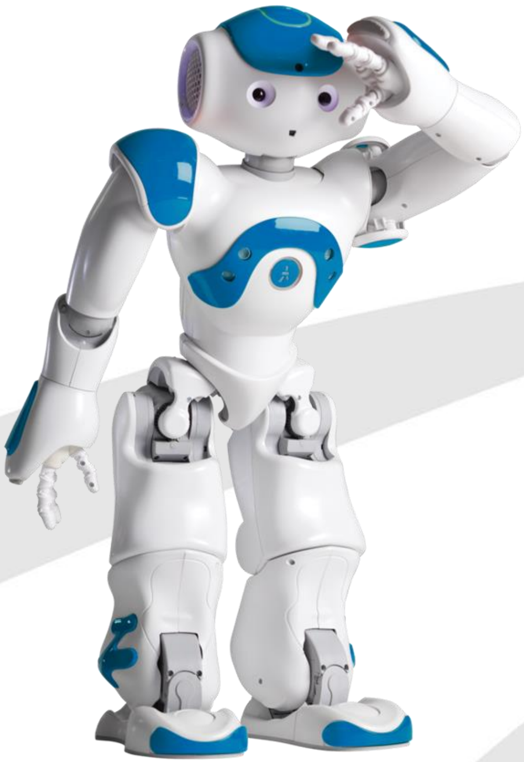
Robotization, computerization, globalization,…
Old jobs disapear, new jobs emerge → Education



Automated production

- Radiotrician

- New is cheaper than repairing; radios, computers, refrigerators,…
- modules; construction,..
- Not just production, also work of secretaries, bookkeeping, logstics, computer programming, …

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Prefab in construction

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Auto Industry

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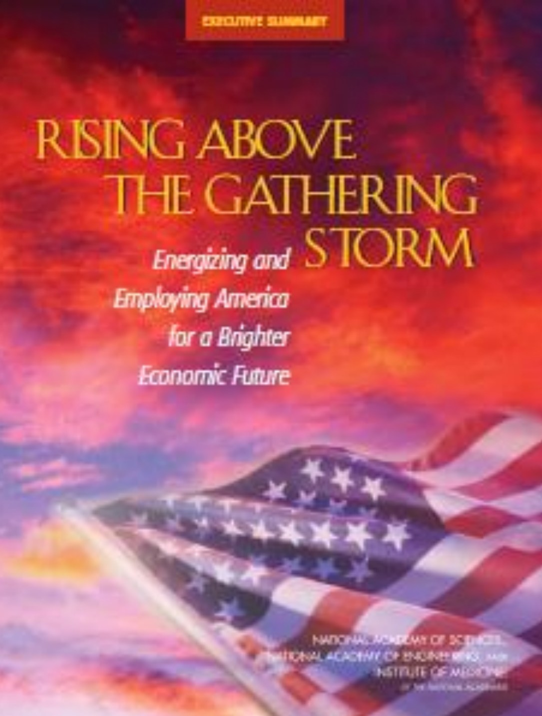
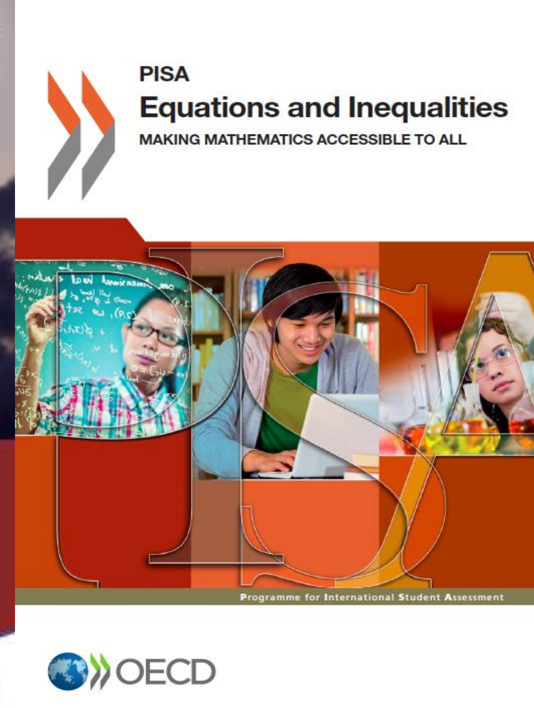
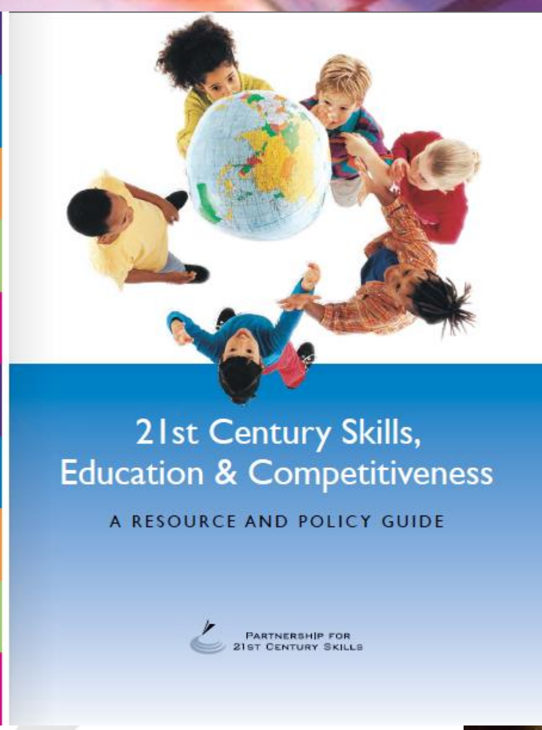
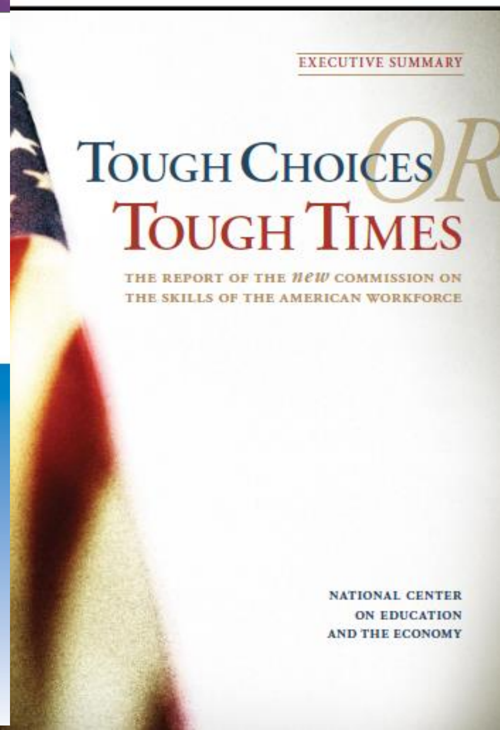
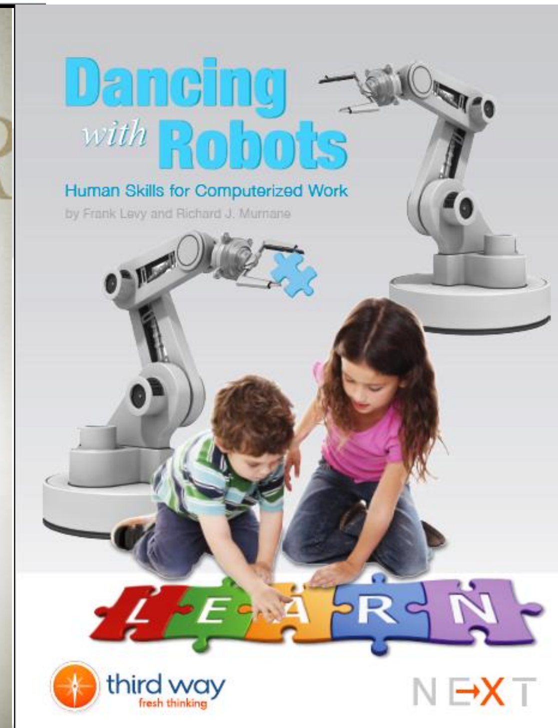
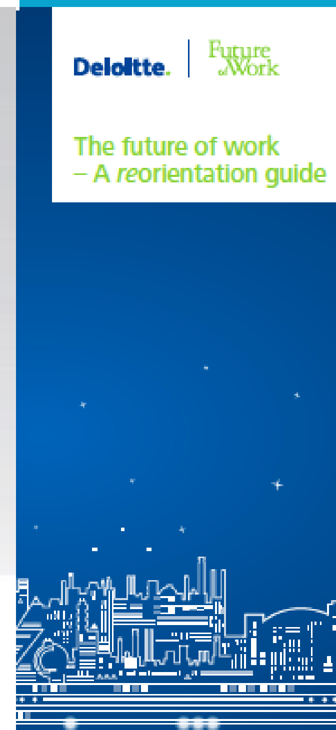

EXECUTIVE SUMMARY
RISING ABOVE THE GATHERING STORM
Energizing and Employing America for a Brighter Economic Future

Policy Information Report
America's Perfect Storm
Three Forces Changing Our Nation's Future
ETS

PISA
Equations and Inequalities
MAKING MATHEMATICS ACCESSIBLE TO ALL
Programme for International Student Assessment
OECD

Department for Business Innovation & Skills
RESEARCH

21st Century Skills, Education & Competitiveness
A RESOURCE AND POLICY GUIDE
Partnership for 21st Century Skills

EXECUTIVE SUMMARY
TOUGH CHOICES OR TOUGH TIMES
THE REPORT OF THE NEW COMMISSION ON THE SKILLS OF THE AMERICAN WORKFORCE
NATIONAL CENTER ON EDUCATION AND THE ECONOMY

Dancing with Robots
Human Skills for Computerized Work
by Frank Levy and Richard J. Murnane
LEARN
third way
fresh thinking
NEXT

Deloitte.
Future of Work
The future of work – A reorientation guide

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Globalization & digitalization

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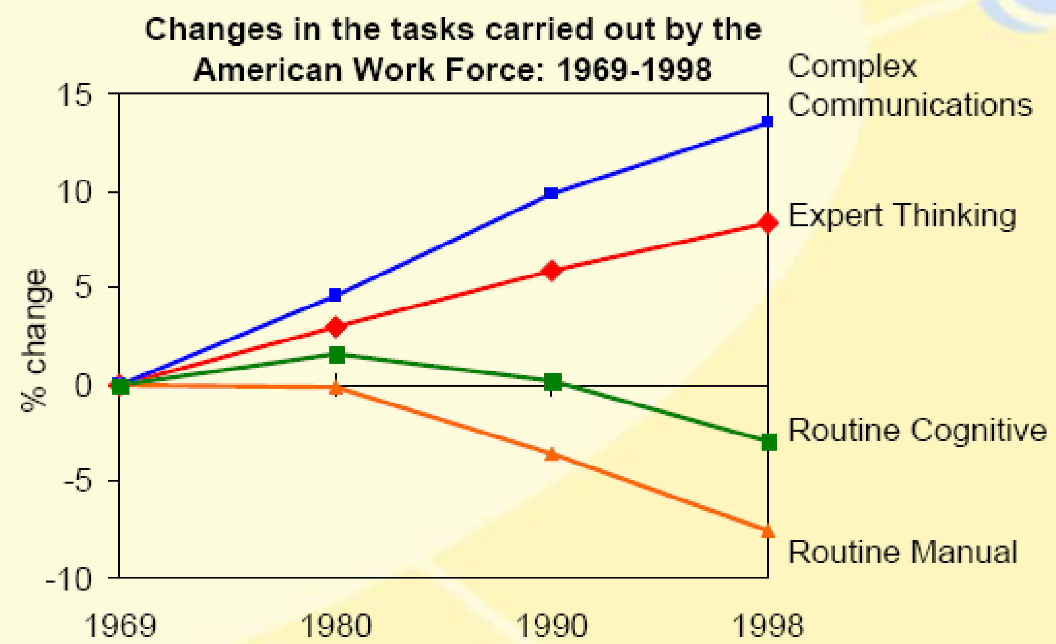
2004
Jobs are
disappearing

The Changing Nature of the Workforce
(Levy & Murmane 2004)

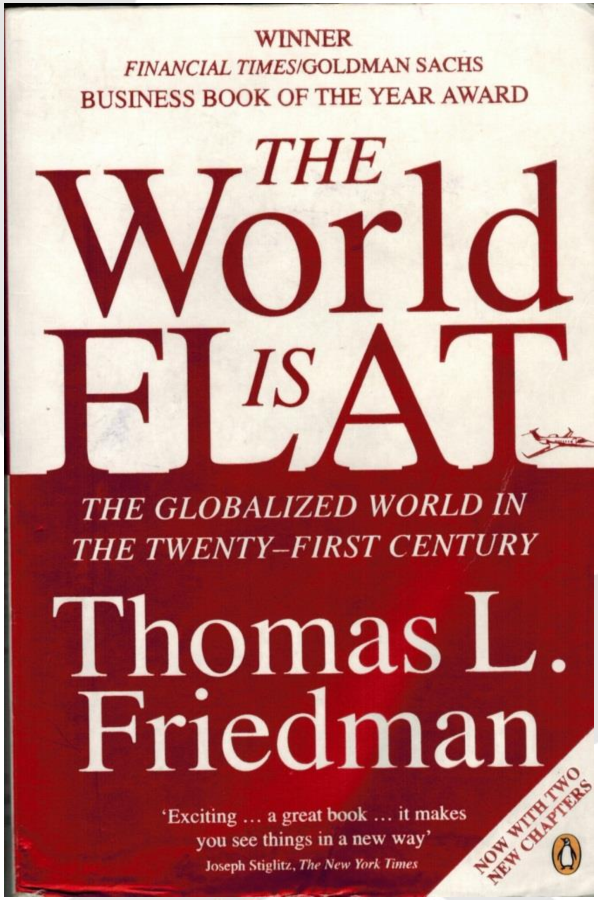


Note: From chapter 3 of Levy- Murnane, *The New Division of Labor* (2004).

Tasks that can be captured in routines disappear

- Jobs that can be broken down into repeatable steps can easily be replaced
- What will be left will be tasks that require flexibility, creativity and life-long learning, and the ability to communicate with people
  - Many jobs are already gone
    - in manufacturing, clerical work, programming
  - The work changes
    - secretaries, bank employees have got more complex tasks
- Factories need people who know how to work with computerized machines

Globalization



The effects of computerization and globalization overlap and reinforce each other.

Routine tasks can easily be outsourced, since information technology enables a quick and easy worldwide exchange of information.

Relation between computer and work

- Replacement:

  The computer takes your job

- Augmenting:

  The computer helps you to do your job ⬄ Need for complementary skills

A split in society

Goos en Manning found a split into *'lousy jobs'* and well-paid *'lovely jobs'* within the UK

The lovely jobs are jobs that require flexibility, problem-solving capabilities, life-long learning and social competencies

Jobs in the middle segment disappear
Problems for Middle Vocational Education

→ **Innovate foundational education**

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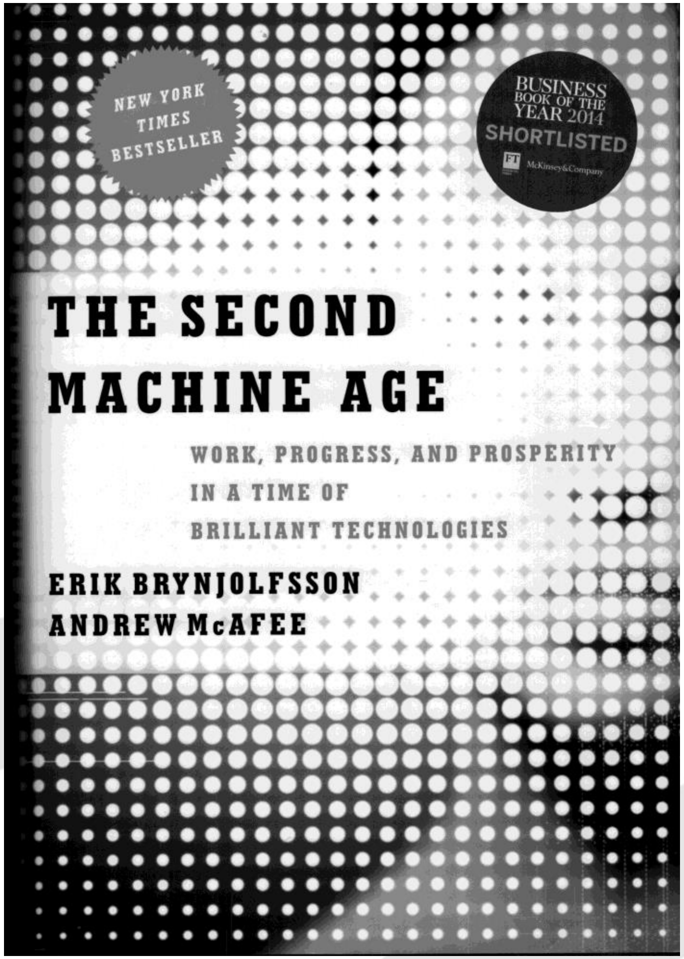
Brynjolfsson & McAfee. (2014) *The second machine age:*



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Digitizing almost everything → miniaturization, dropping prices

- Moore's Law: Every two years, the number of transistors on a chip double, and the prices halve
- Digitizing: transforming all sorts of information into one and zero's:

  text, sound, photos, videos, sensor output, ..

- Smaller, cheaper, all sorts of new combinations

Digitizing, miniaturizing, price drop

- Google's self-driving car contains 64 lasers and 64 sensors in a unit that rotates 10 ten times per second. This generates 1.3 million data points which are processed real-time to create a 3D image that extends in all directions for 100 meter.

- In 2000 a similar system would have costed about 35 million dollar, in 2013 about 80 thousand dollar.

21^{st} century skills

21st century skills

Many projects with similar esults (Voogt & Pareja Roblin, 2010):

- Partnership for 21st Century Skills
- EnGauge
- Assessment and Teaching of 21st Century Skills
- National Educational Technology Standards (NETS)
- Technological Literacy for the 2012 National Assessment of Educational Progress (NAEP)
- Studies carried out by the European Union, OECD and UNESCO

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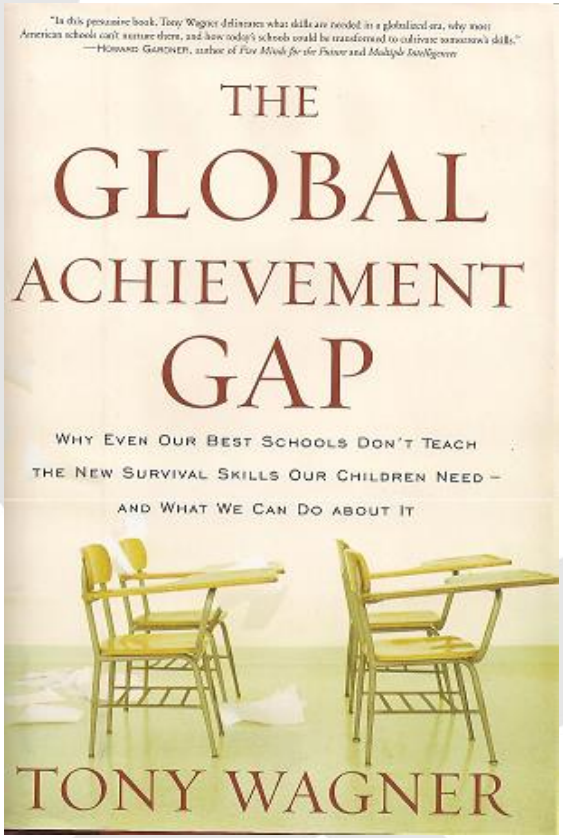
Gap between schools and society

Even our best schools don't teach the skills our children need in the 21st. Century



Expectations of CEO's are very different from what schools offer: *"First and foremost, I look for someone who asks good questions."*

Learning to think not a school goal.

School Goals ⇔ Expectations of the wider society

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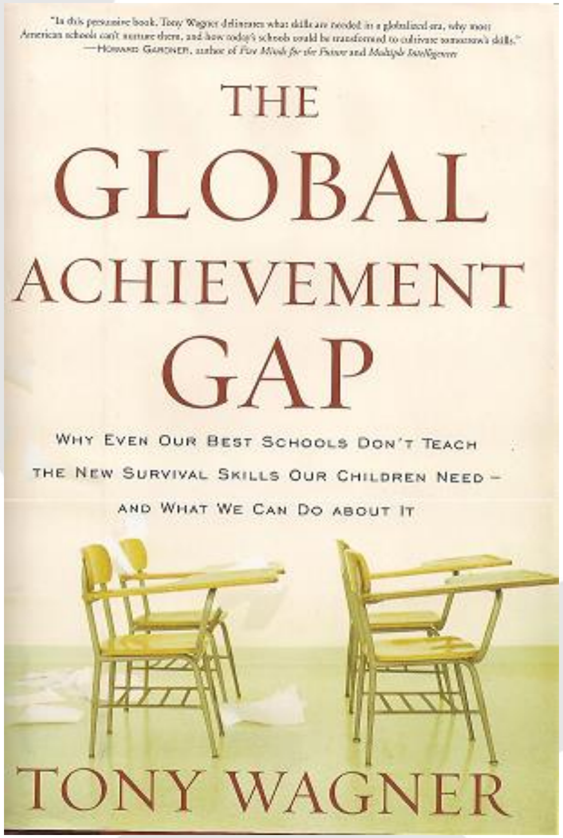
Gap between schools and society



21st. Century skills:

- Critical thinking and problem solving
- Collaboration across networks
- Agility and adaptability
- Initiative and entrepreneurism
- Effective communication
- Accessing and analyzing information
- Curiosity and imagination

- ICT-literacy

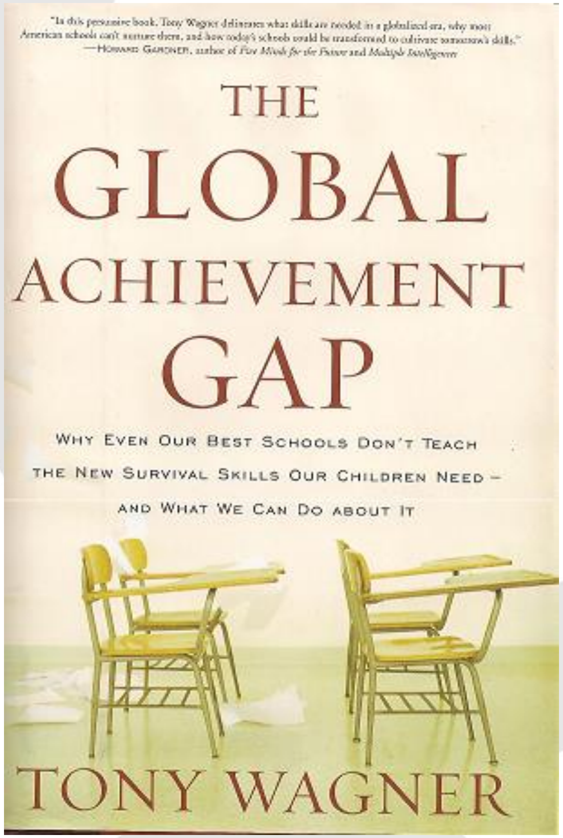
computerized circuit design

- task: laying out printed circuit boards.
- input: drawings
  - circuit board’s required dimensions
  - location of the critical components
  - specifications for each component
  - rough schematics that describe connections
- using computer assisted design tools
  - design the circuit board’s layout
  - number of layers
  - electrical paths
  - location of the components
- output:
  - blueprint + digitized machine settings

Process

- input does fully describe what the client wants
  → asks colleagues for help to fill in the gaps
- two components too close to each other
  → fixed
- the circuit board needs 4 layers instead of 3; connections that can be drawn, are impossible to fabricate
  →face-to-face interaction with client ⇔ common understanding
- last-minute modifications

“Jury-ready” :



Would they know how to distinguish fact from opinion, weigh evidence, listen with both head and heart, wrestle with the sometimes conflicting principles of justice and mercy, and work to seek the truth with their fellow jurors?”

General Educational Goals

- Focus on
  - problem solving
  - critical thinking
  - working in groups
  - communicating
- What does this mean for mathematics education?

Interactive and problem-centered mathematics education

- Reform mathematics (NCTM) points to this very type of activities—problem solving, collaborating, communicating etc., as necessary conditions to enable students to construct meaningful mathematics.

- However, creating mathematics classrooms which can be characterized as interactive and problem-centered is not easy. (Has been underestimated)

Fostering reform mathematics

- Establish an inquiry classroom culture
- Design instruction that enables students to construct mathematics
- Cultivate student motivation

Establish an inquiry classroom culture

Shift from
conventional school-math classroom norms to
inquiry-oriented classroom norms

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Establish an inquiry classroom culture

Shift from
conventional school-math classroom norms to inquiry-oriented classroom norms

Conventional school-math classroom norms encompass, the obligation to:

- adapt to the teacher's way of reasoning,
- follow given procedures,
- not trust your own reasoning,
- look to the teacher for the right answers

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Establish an inquiry classroom culture

Shift from
conventional school-math classroom norms to inquiry-oriented classroom norms

<u>Inquiry classroom norms</u> encompass, the obligation to:

- explain and justify one's solutions,
- try and understand other students' reasoning,
- ask questions if you do not understand, and
- challenge arguments you do not agree with

Cultivating an inquiry classroom culture

Teacher's role:

- Asking for explanations
- Asking for clarifying questions
- Passing the problem along
- Asking for a personal judgment
- Promoting that students listen and try to understand

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Design instruction that enables students to construct mathematics

- Focus on the mental activities of the students
  - Anticipate how those mental activities might help them to develop mathematical insights (Simon, 1995)
- Fitting tasks
- Instructional Design; Instructional sequences/ Learning progressions
- Instruction Theory

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Cultivate student motivation

- *Task orientation* implies that the student's concern is with the task itself, and on finding ways of solving that task.
- *Ego orientation* implies that the student is very conscious of the way he or she might be perceived by others. (Jagacinski & Nicholls, 1984)
- *Cultivating task orientation:*
  - creating a classroom culture, where students measure success by comparing their results with their own results earlier.

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Cultivate student motivation

- Mathematical interest:
  - In order to be able to construct mathematical concepts and tools, students have to reflect upon the mathematics involved:
    - Is this always true?
    - Can I adapt or improve this method?
    - …

Mathematical Goals

What Mathematics should we aim for?

- For now, I will first focus on the preparing for the workplace
  - Discuss Math for everyday life later on
  - Other important considerations
    - Further education
    - Math as cultural heritage

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What Mathematics should we aim for?

1. *Context (the workplace)*
   – The character of mathematics in the workplace
2. *Competencies*
   - Mathematical competencies that complement the work of computers
3. *Content*
   – Mathematics that gains importance in the digital society

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1. Context: The character of mathematics in the workplace

- Applied problems that have practical implications; room for several solutions and trade-offs
- Often implicit, using idiosyncratic solutions, shaped by practices and tools
  - A majority of nurses are unsuccessful on school-based proportion tests (Perlstein, Callison, White, Barnes, & Edwards, 1979)
  - However, they are 100% accurate when administering medicines in correct ratios, using medicine-specific rules of thumb (Hoyles, Noss, Kent, & Bakker, 2013)

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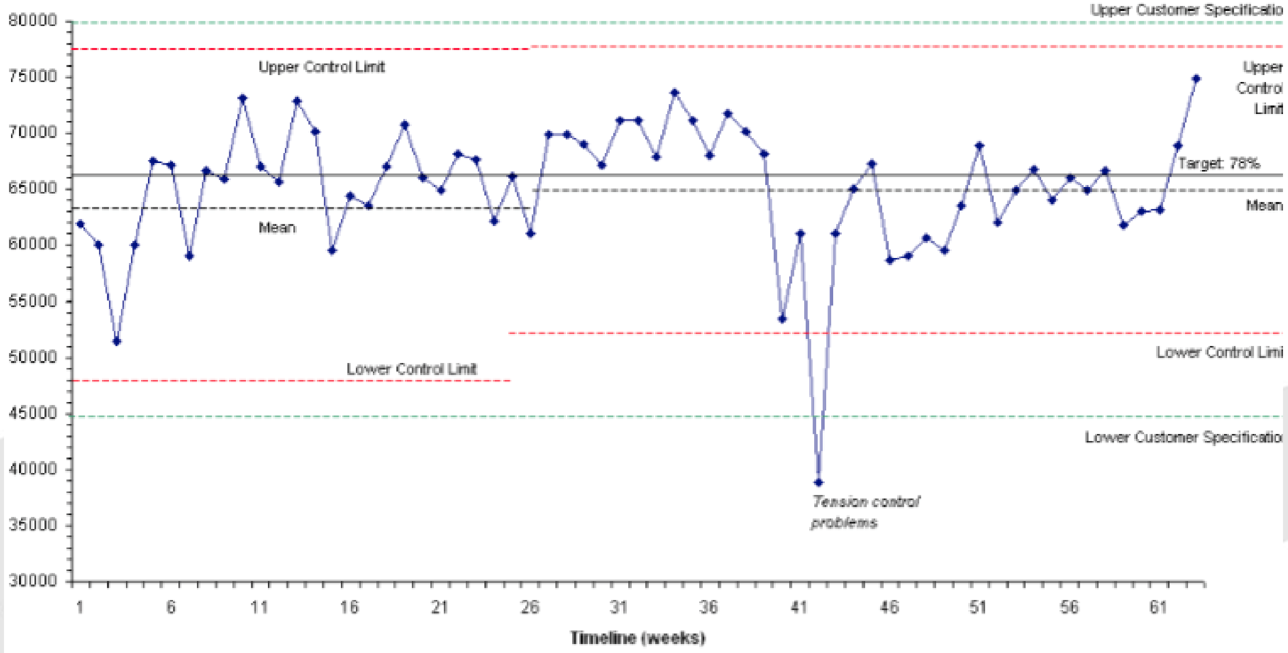
Techno-mathematical Literacies

Idiosyncratic forms of mathematics that are shaped by workplace practices, tasks, and tools; a combination of mathematical knowledge and contextual knowledge.

(Hoyles and Noss, 2003)



Control chart of production of adhesive labels on large rolls

The character of mathematics in the workplace

- Change over time: From manual tools and mechanical machines towards automated machines, embedded software and computer tools

➢ Need for a global understanding of the hidden mathematics & being able to communicate about this mathematics with colleagues and customers

2. Competencies: Mathematical competenties for the digital society

- Adapt mathematics education to the requirements of acting in an environment where computers do all calculations

- Focus on competencies that allow one to use computers in a sensible and productive manner

“Because of computerization, the use of abstract models now permeates many jobs and has turned many people into mathematics consumers” (Levy en Murnane, 2006)

- spreadsheets,
- automatics cashiers
- automated production lines

People who use these systems are expected to make decisions on the basis of the output of the hidden mathematical calculations.

- A manager of a clothing store who uses a quantitative model to predict the future dress demand
- A truck dispatcher who uses a mathematical algorithm to determine delivery routes
- An employee of a bakery who monitors the production of bread by means of digital data instead of the smell or looks of the bread.

If the decision maker does not understand the underlying mathematics, he or she is very vulnerable to serious errors of judgment.

Stephen P. Keeler & Thomas A. Grandine
Boeing Applied Mathematics Group

In school the professor formulates the problem and you solve it—you hope. In industry, you formulate the problem and the software solves it—you hope.

Mathematical competenties for the digital society

- Two roles of computers:
  - Replacement; tasks being taken over
  - Supplementary; computer as a tool
- Do not focus mathematics education on what computers can do better; focus on what is needed for working in a computerized environment

Conrad Wolfram

Wolfram Group
Mathematica
Computerbased Math

Doing mathematics:

1. Looking where mathematics is applicable
2. Translating a question or a problem into a mathematical problem
3. Solving the mathematical problem
4. Translating the solution back, and evaluating the solution

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Wolfram Group
Mathematica
Computerbased Math

Doing mathematics:

1. Looking where mathematics is applicable
2. Translating a question or a problem into a mathematical problem
3. <u>Solving the mathematical problem</u>
4. Translating the solution back, and evaluating the solution

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Working in a computerized environment

- *Recognizing* problems that can be solved mathematically
- *Translating* problems in such a manner that they can be processed by computers
- *Understanding* the mathematics that is involved
- *Interpreting* and *evaluating* output

2. Mathematical competenties for the digital society

Recognizing, translating, understanding, interpreting and evaluating → competencies in the domains of

a. Applying & Modeling
b. Understanding
c. Checking

2a. Applying & Modeling

- Construct models; relation reality ⇔ model
  - Often complex ill-defined problems, with varying conditions, trade-off costs and benefits, optimalization

- Modelers have to connect mathematical concepts and procedures with applications—which often is a problem with school mathematics →

"Teaching mathematics as to be useful"

(Freudenthal, 1968)

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2a. Applying & Modeling

- Competencies concerning the creation of models,
  - interpreting and structuring problem situations
  - translating them into mathematical models
- Competencies concerning the result of mathematical elaborations,
  - interpreting, describing, explaining, and evaluating the outcomes of the calculations implied by models

! Judging the results of modeling activities carried out by others

PISA 2015
Mathematics framework
Modeling capabilities

- transition from reality to mathematics:
  - decoding and interpreting
  - structuring and conceptualizing
  - making inferences and assumptions
  - formulating a model

- translating mathematical results to reality:
  - summarizing and presenting results
  - providing explanation or justification
  - interpreting or evaluating the mathematical outcome

- plus:
  - using graphs, tables, diagrams, pictures, equations and formulae.

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2b. Understanding

- Workers will have to understand the mathematics underlying the work the computer does—if only to communicate with co-workers and clients

  (Kent, et al., 2007; Gainsburg, 2007)

- Conceptual understanding
  - Relational understanding ⇔ Instrumental understanding (Skemp, 1978)

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Understanding the key underlying ideas

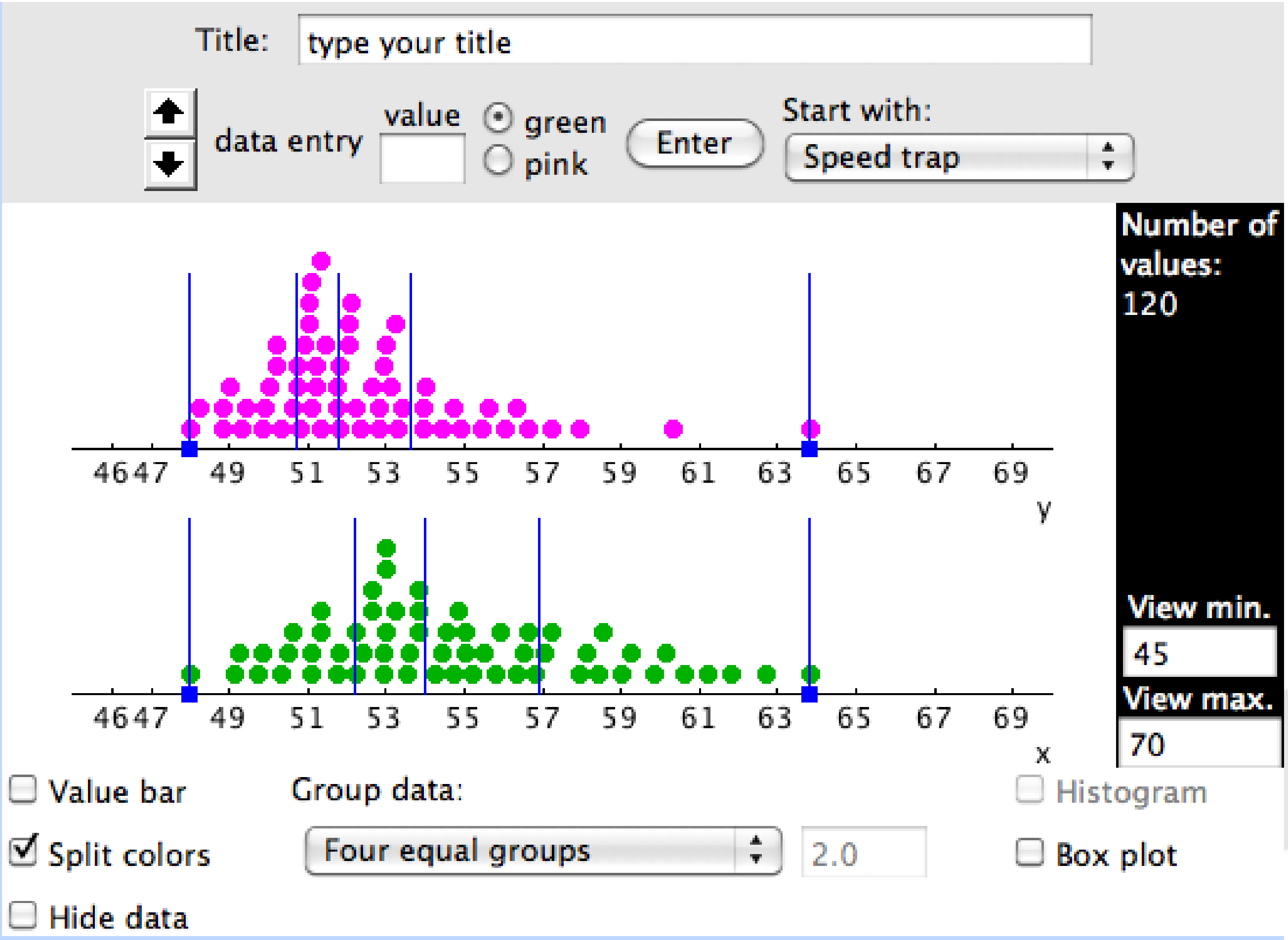
- More people will have to understand more mathematics
- Most people do not need to understand the formalisms, but have to understand the key underlying ideas (Kaput, 1997; Hoyles et al., 2010)
- e.g. integral and differential calculus:
  - ideas of rate of change, accumulation, variable rates ⇔ accumulation, and approximation

Democratizing access to the mathematics of change and variation

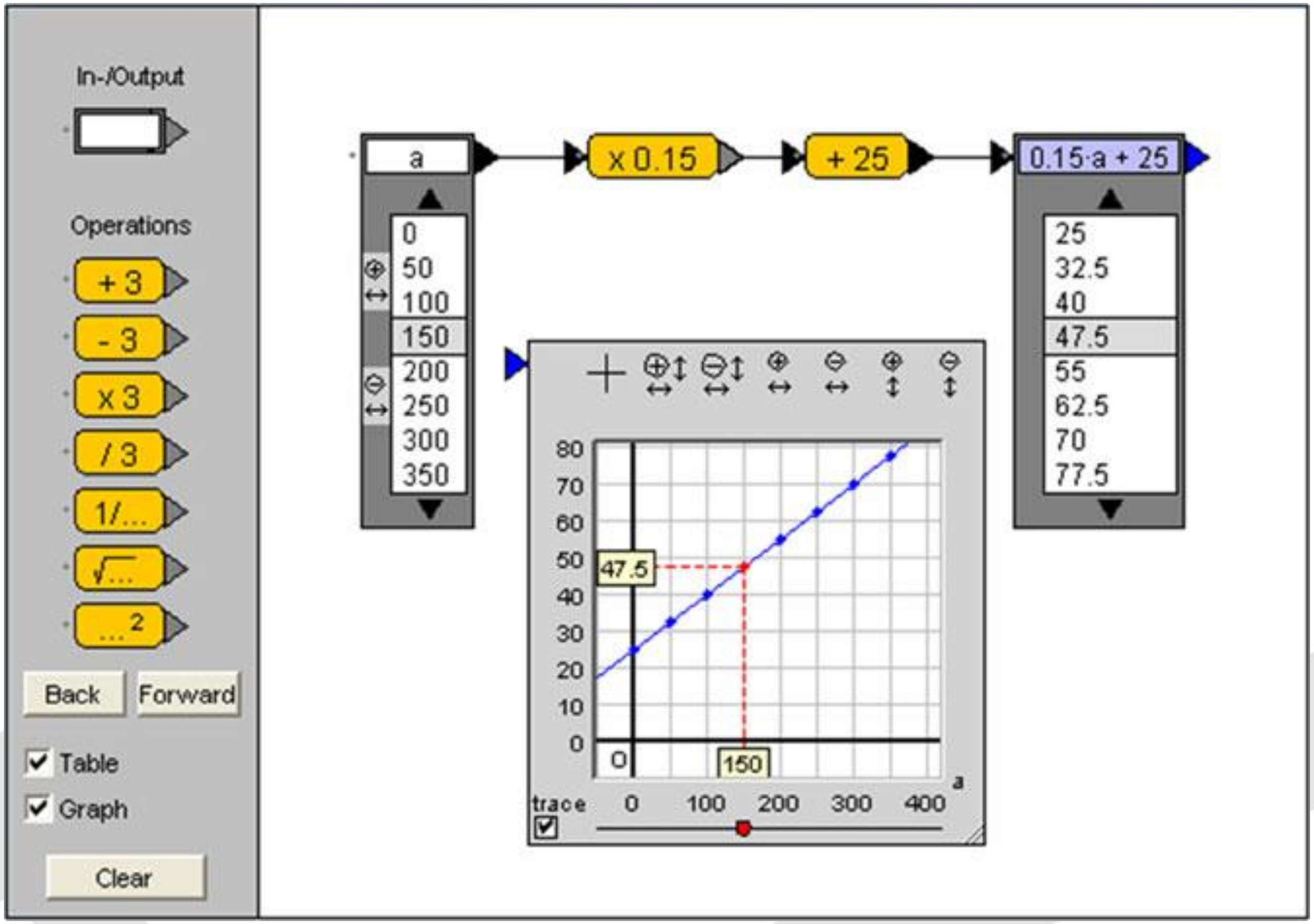
Didactical, dynamic computer tools

- Computer tools that are tailor-made to support the development of coming to grips with the key ideas in specific areas.

Data Analysis

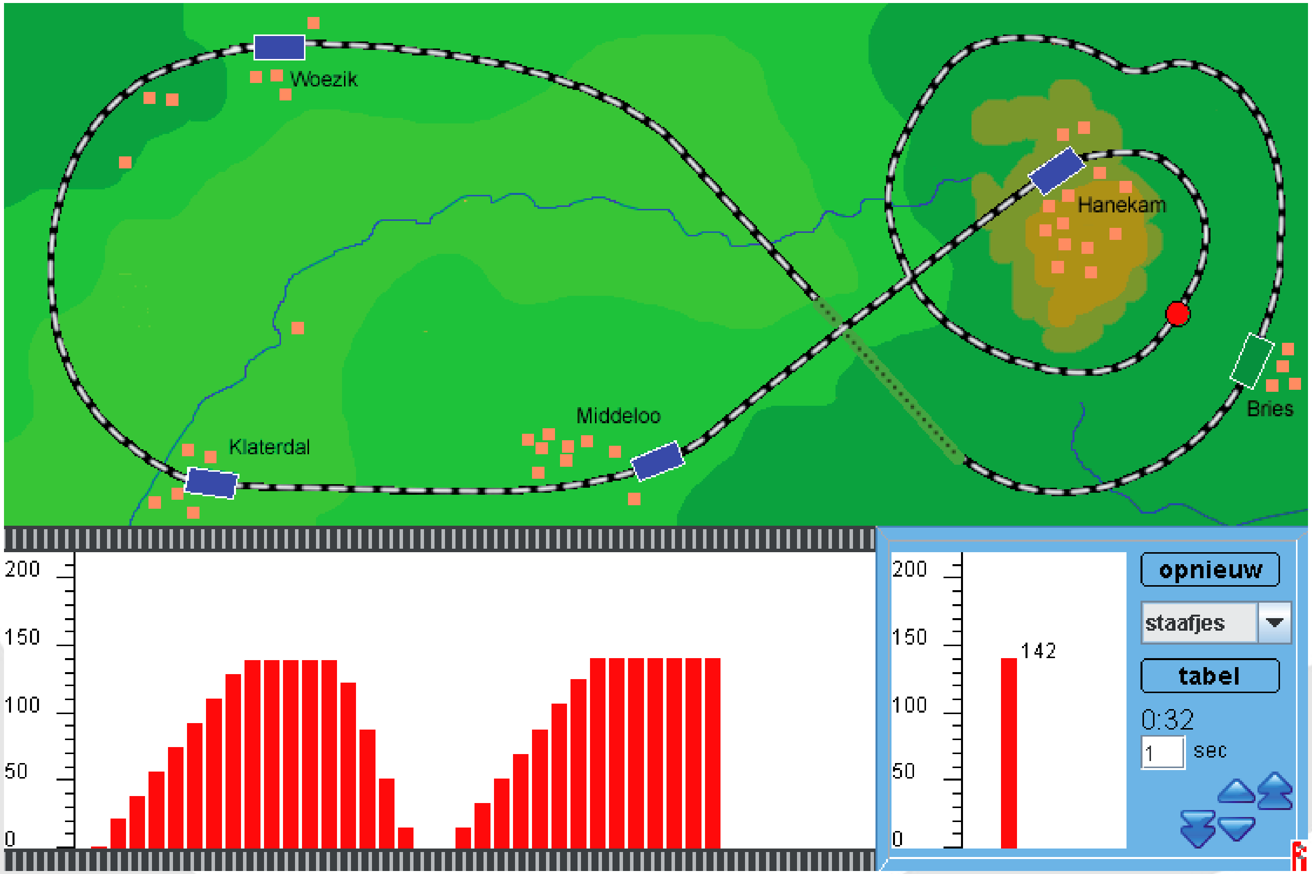


Functions



In-/Output
Operations
+ 3
- 3
x 3
/ 3
1/...
√...
...2
Back
Forward
Table
Graph
Clear
a
x 0.15
+ 25
0.15·a + 25
0
50
100
150
200
250
300
350
25
32.5
40
47.5
55
62.5
70
77.5
80
70
60
50
40
30
20
10
0
47.5
150
trace
0
100
200
300
400
a

Integral Calculus



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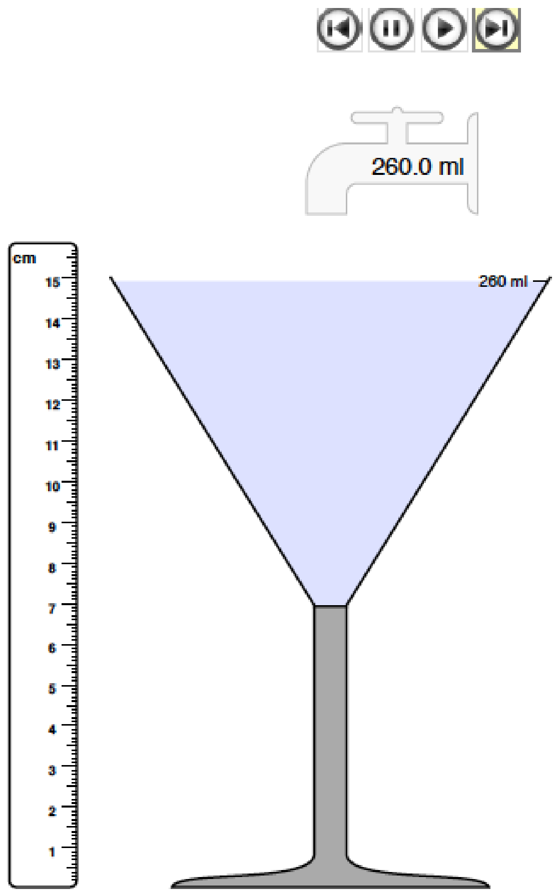
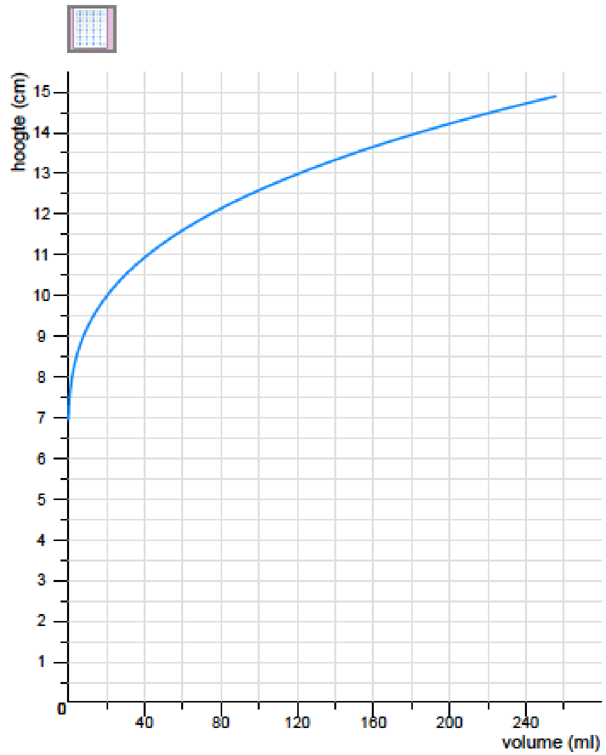
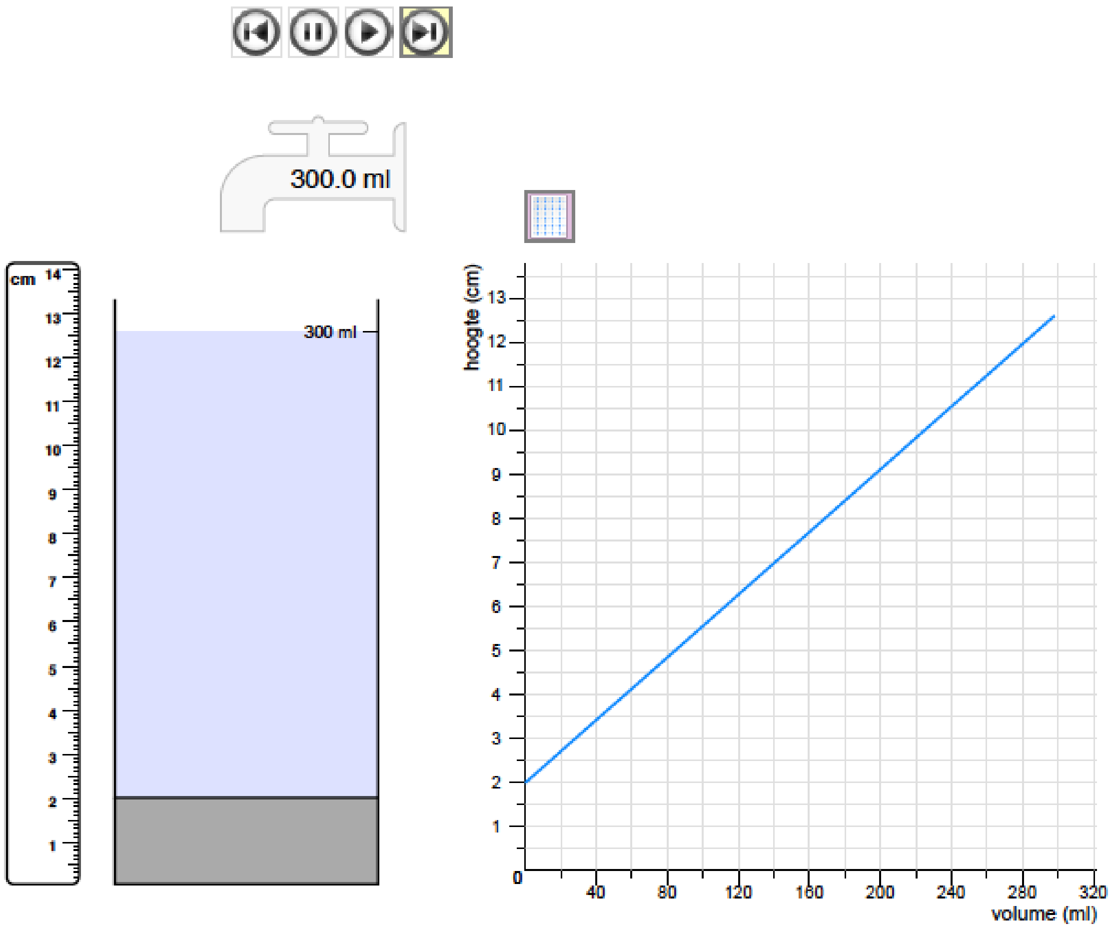
Differential Calculus

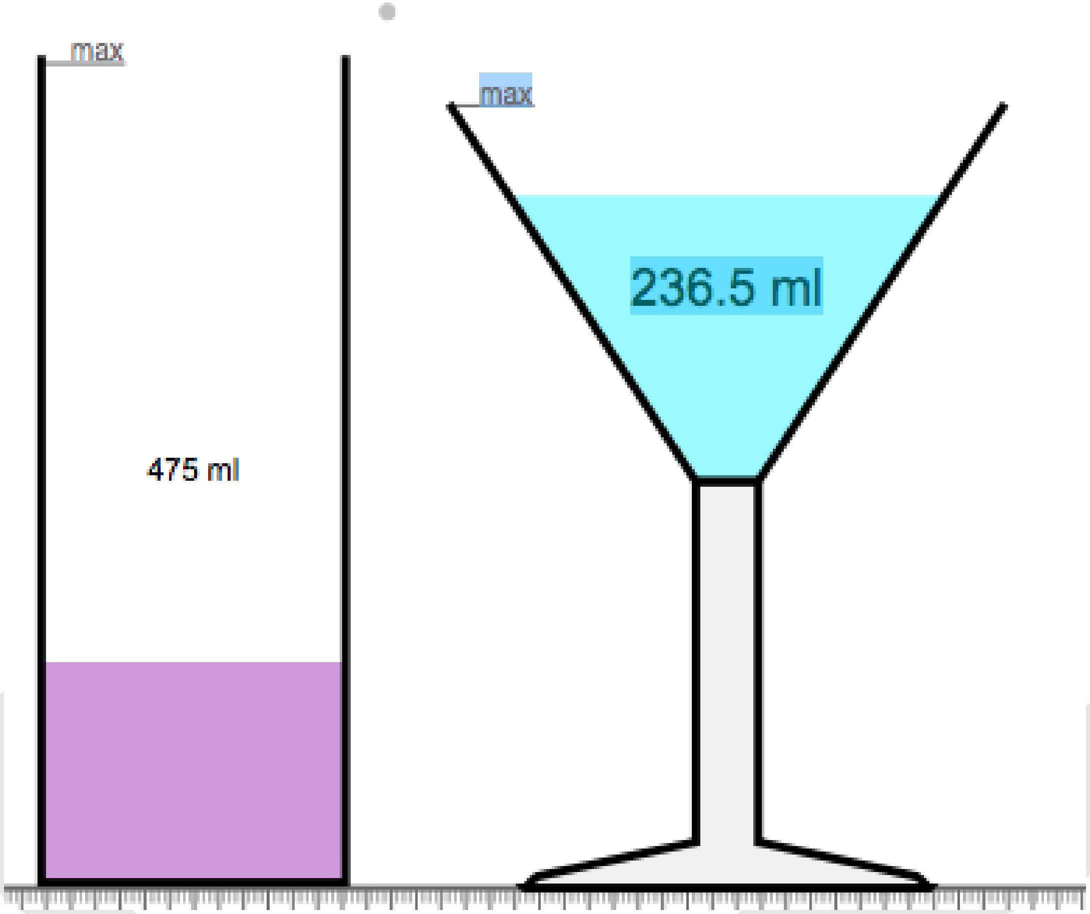




Differential Calculus



Differential Calculus



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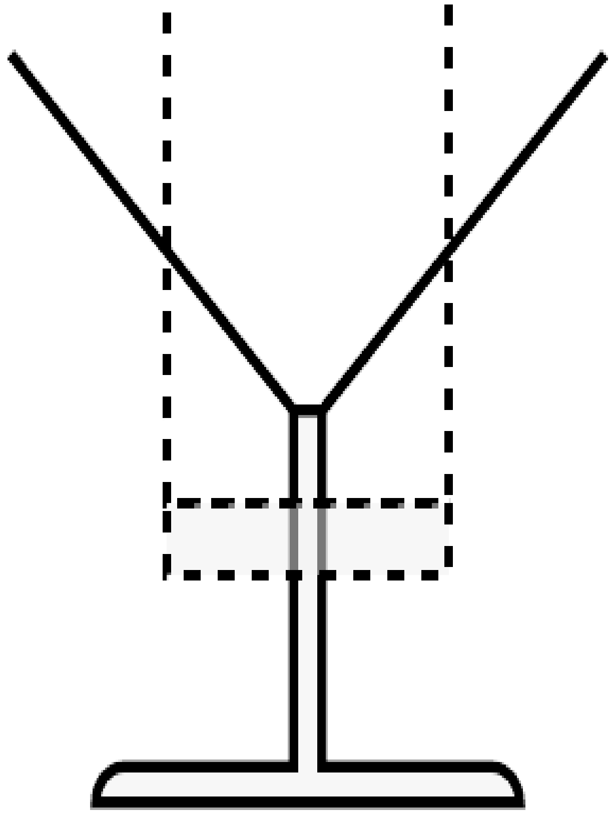
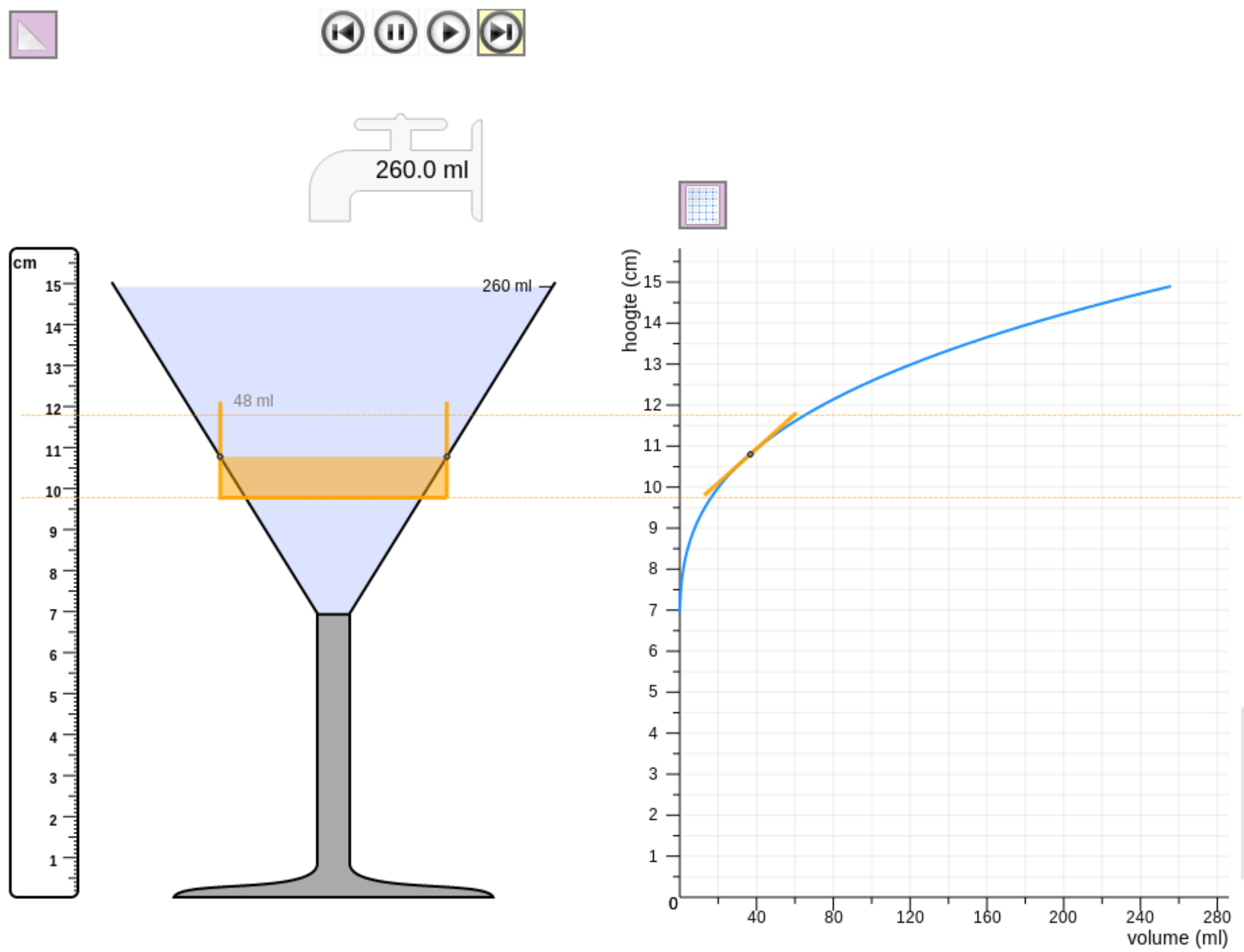
Differential Calculus



Differential Calculus



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Ready-made computer tools

- Use of computer tools; hidden, or in the form of calculators, spreadsheets, Computer Algebra Systems, graphing tools, etc.

- Instrumentation (Drijvers & van Herwaarden, 2000):
- The user of a computer tool tool has to develop an *instrumentation scheme*, which not only consists of a series of actions but also involves mathematical objects, and strategies.

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2c. Checking the output

Simple arithmetical example

4x27= ?

- Roughly: 4x25=100
- Less than 4x30=120
- Twice 54=108

Number relations: for instance multiples of 25, 75, 125 and so forth ⇔ decimals, fractions and percentages (0,25; ¼, 25% etc.);

- Reasoning that 4x1,25=5, because 4x25=100 thus 4x125=500, or, because 4x1,25 equals 4x1¼.

The basis for global arithmetic

Networks of number relations, e.g.:

- Number relations with 25 as key number:
  - Multiples of 25 and of 125,
  - relations with 1/2, 1/4 and 1/8; 0.5, 0.25and 0.125 ⇔ 50%, 25%,12.5%, etc.
  - Combined with powers of 10
- Number relations involving 24
  - Multiplications with 24 as outcome
  - Such as 3 x 8, 4 x 6, 2 x 12 en 8 x 3, 6 x 4, 12 x 2, but also 2 x 3 x 4 = 24 and 2 x 2 x 2 x 3)

The basis for global arithmetic

- Arithmetical properties
  - Commutative property

    e.g. 25 x 4=4 x 25
  - Associative property,

    e.g. 81 + 37 + 19 = (81+19) + 37
  - Distributive property

    e.g. 22 x 15 = (20 x 15) + (2 x 15)
  - Etc.

Global arithmetic ⇔networks of number relations

→ developing the ability and attitude of checking the output requires a different curriculm content and curriculum structure

(not just for arithmetic)

3. Mathematical content

- Measurement, data collection, variables and co-variation, reading and interpreting data, graphs and charts. (Hoyles, Noss, Kent & Bakker, 2010)
- co-variation and functions, which they see as building blocks for modeling aspects of systems

  (Brady et al, 2015)
- ***Statistics***, *“big data” & statistical literacy* (Gal, 2002).
- ***Space-geometry*** ⬄ CAD CAM, 3D imaging and 3D printing.

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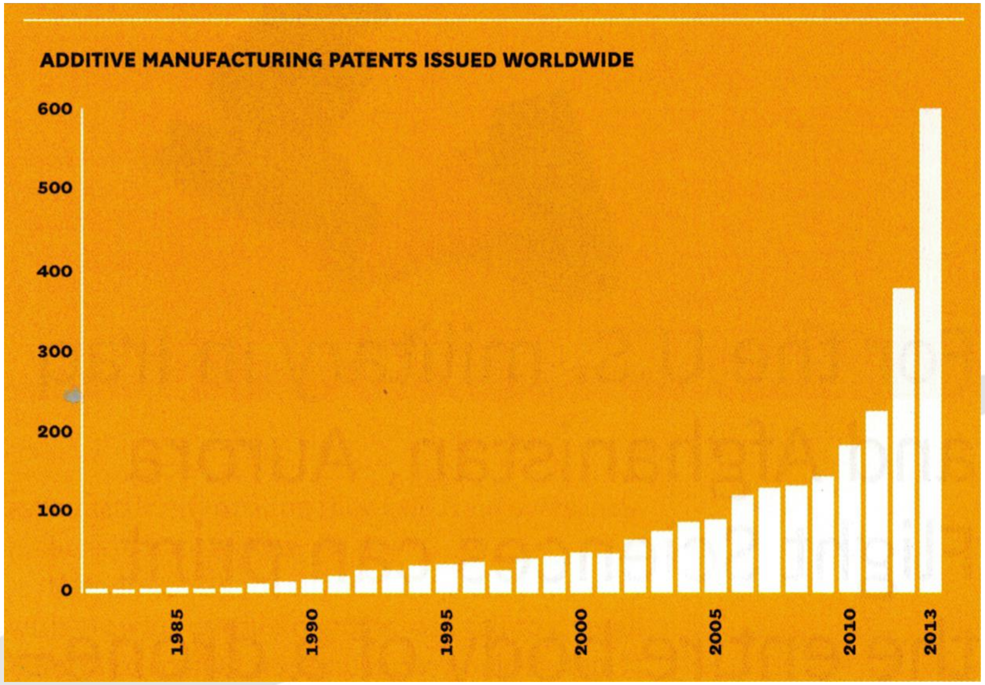
Number of patent applications on 3D printing



1980 -- 2013

3. Mathematical content

- Computers use mathematical models of reality, which consist of systems of interconnected mathematical relations

  → variables, co-variation, and functions.

- Phenomena from reality are translated into numerical quantities

  → understanding of the process of quantifying reality

  → a broad understanding of measuring;

  - notions of uncertainty and repeated measurement, mean, and measurement error
  - data creation and sampling come to the fore

Mathematics for everyday life

- Demands grow as a consequence of the increasing digitalization of our society.
- Importance of quantitative literacy for democratic discourse and civic decision-making (Steen, 2001)
  - interest-rate cuts by the Federal Reserve, changes in gasoline prices, trends in student test scores, election results, and risks of dying from colon cancer

Mathematics for everyday life

- Large overlap between preparation for work or for everyday life.
- Important goals here are,
- self-reliance and self-confidence when dealing with mathematics in everyday life, and active citizenship.
- Importance of one of the key 21st century skills: critical thinking.

ALL Numeracy Framework

Quantity and Number

- length area, weight, time, money etc., but also humidity, air pressure, population-growth rates, and profits

Dimension and Shape

- real objects and abstract things, and visualizations thereof (maps, projections, etc.)

Data and Chance

- variability, sampling, error, prediction, signal and noise; data collection, data displays

ALL Numeracy Framework

Patterns, Functions and Relationships

- using tables, graphs, symbols and words

Functions and Relationships Between Variables

- understanding (basic) economic, political and social analyses

Change & Rates of Change

- how organisms grow, populations vary, prices fluctuate, and traveling speed may vary, compound interest

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Concluding remarks

Globalization and digitalization require us to reconsider the content and goals of mathematics education

The focus will have to be on foundational education

- Digitalization especially affects jobs at the middle and the bottom of the labor market
- We have to make sure that students that may leave school at an early age get a sound preparation.

Thank you!