



Energizers and hindrances for the design and implementation of mathematics curricula

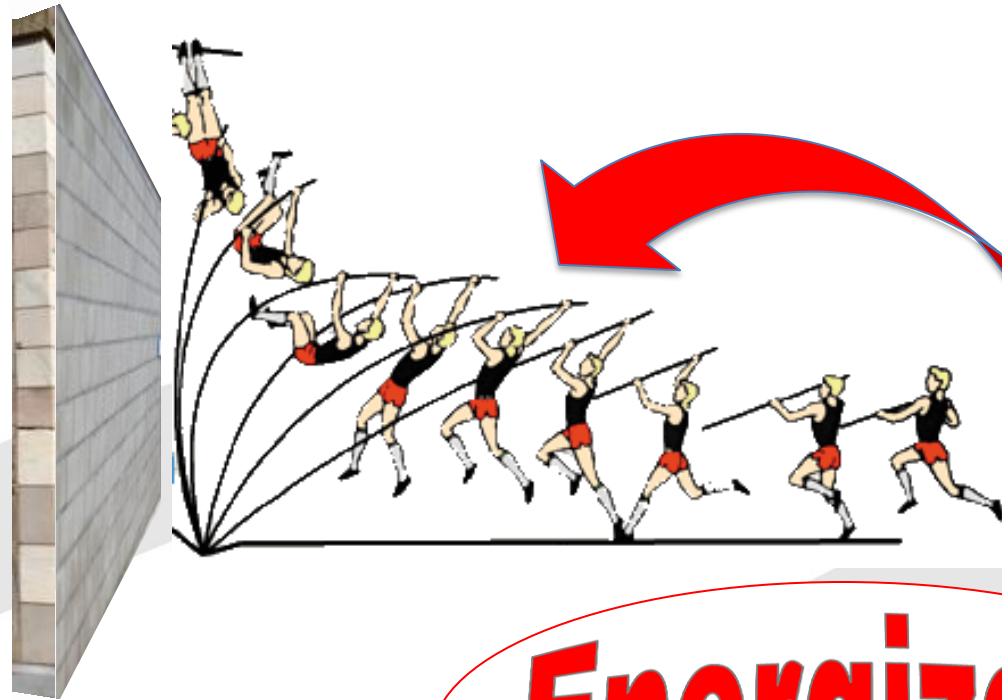
Ferdinando Arzarello

Dipartimento di Matematica “G. Peano”

Università di Torino

A metaphor for my talk

Hindrances



Energizers



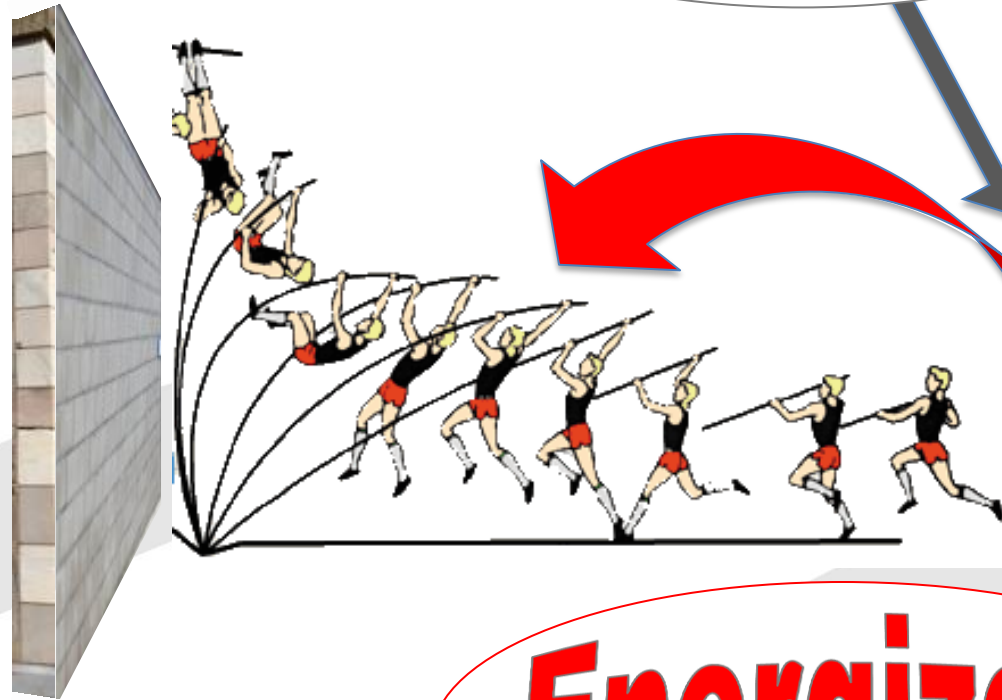
REPÚBLICA
PORTUGUESA

EDUCAÇÃO

A metaphor for my talk

Hindrances

Ambiguities



Energizers

Overview

XXI century society: a challenge for curricula

A multidimensional **framework** for curricula

Energizers for acting on teachers' beliefs:

Technology

Assessment

Teachers' education programs

Hindrances for learning:

Technology

Gender gap in Mathematics

Paideia 2.0: an example

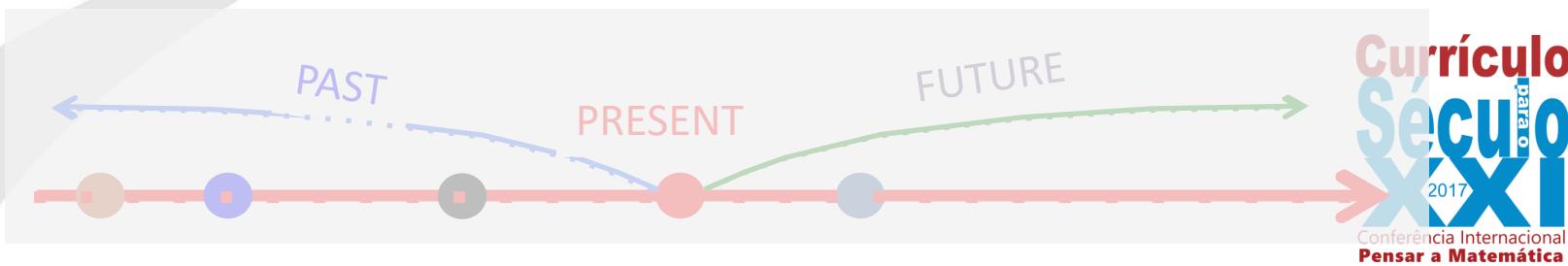
XXI century society: a complex challenge for curricula

ZYGMUNT BAUMAN Liquid Times Living in an Age of Uncertainty



Liquid Times:

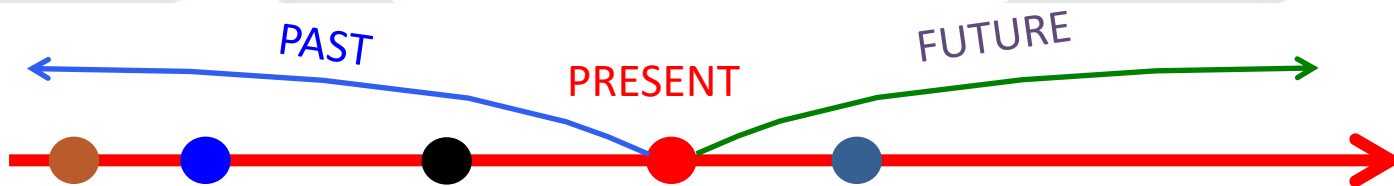
- Constantly changing conditions
- Uncertain future
- Collapse of long-term thinking
- Focus on short-term goals
- Focus on individual responsibility
- Risk is to stand still



- *Pointillist time*
- *Nowist culture*: dissolution of the plot that connects the present with the past and with the future
- *Lack of narratives*: the fragments threaten to become hegemonic.

Towards a παιδεία 2.0 (= paideia + “liquid” practices)

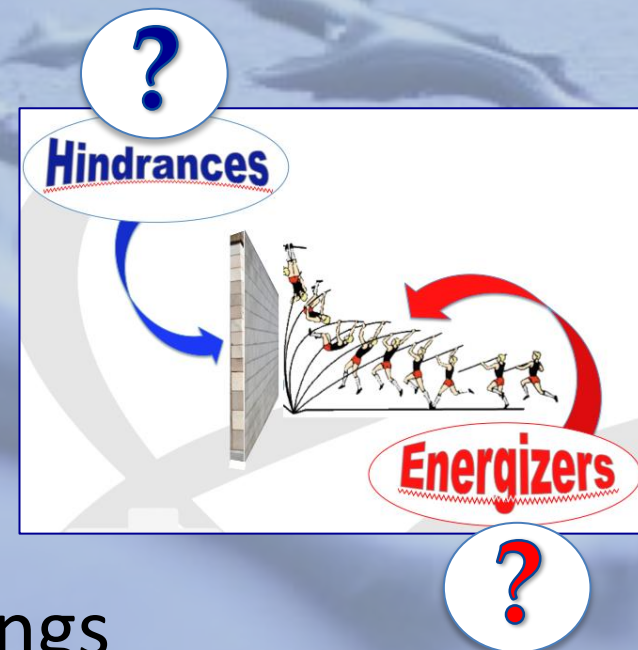
- Against pointillist time
- Reconstructing narratives
- Reconstructing intersubjective links
- Using the ICT affordances for surfing through the liquid society



Liquid practices

ICT (e.g. Mobiles) affordances
to enable learning in liquid society:

Accessing
Sharing
Building
Co-creating
Supporting
Managing
Across settings
Across time



(Franziska Trede, 2016)



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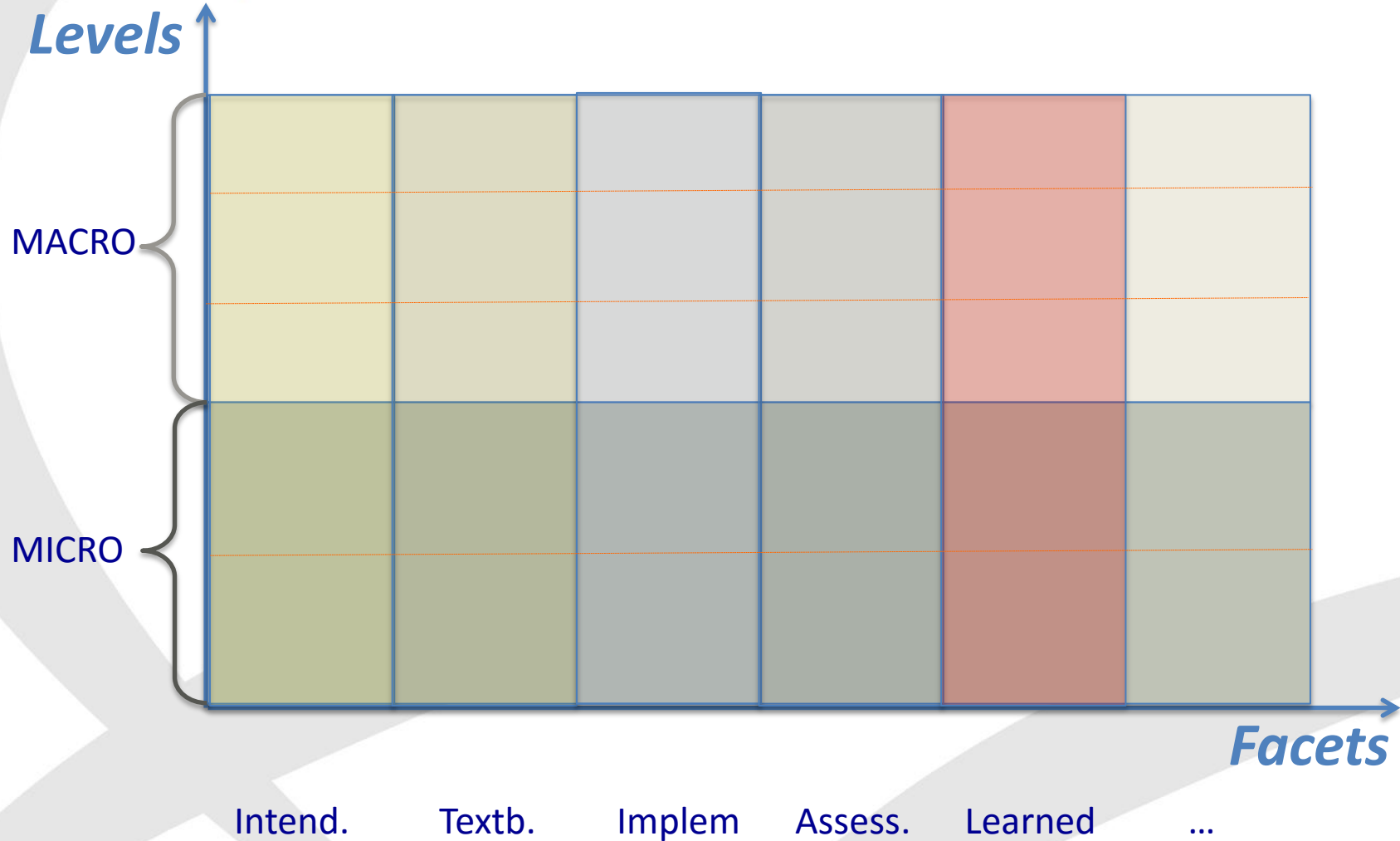
Hindrances for learning:

Technology

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Paideia 2.0: an example

A multidimensional Framework



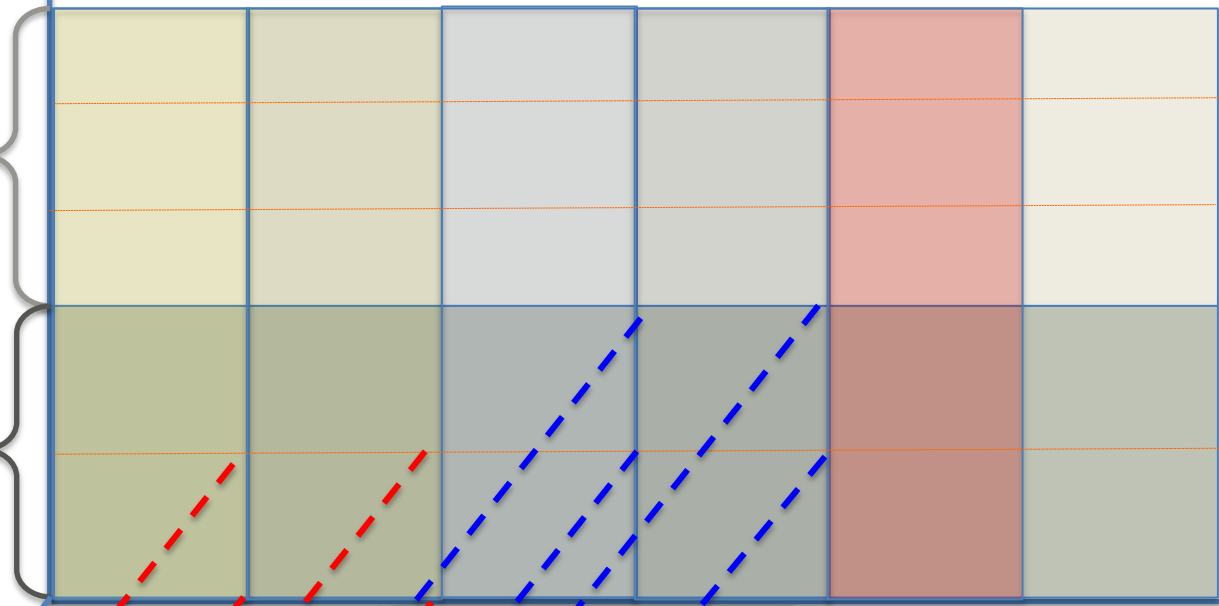
A multidimensional Framework



Levels

MACRO

MICRO

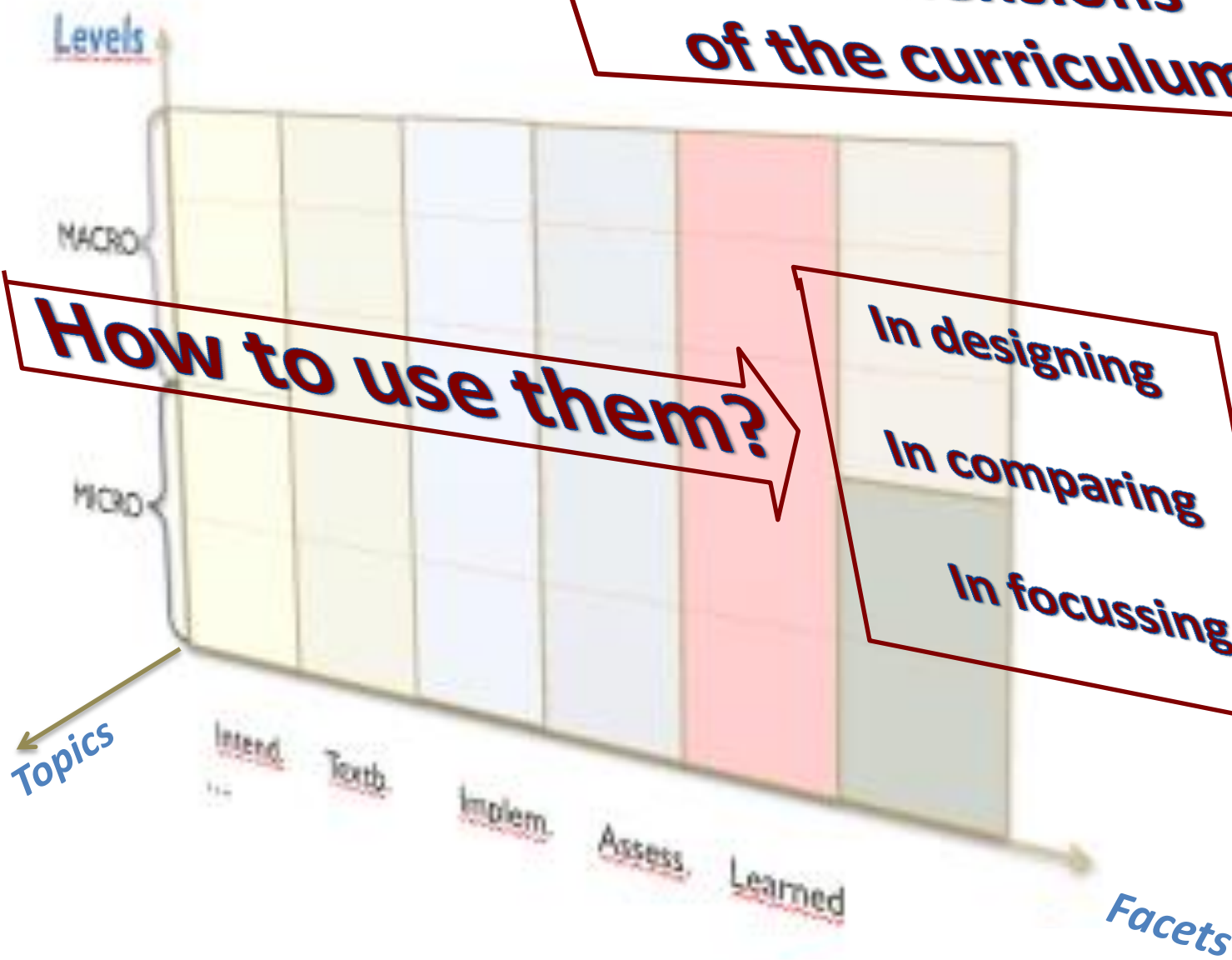


Intend. Textb. Implem. Assess. Learned ...

Facets

Topics

The dimensions of the curriculum



How to use them?

- In designing
- In comparing
- In focussing

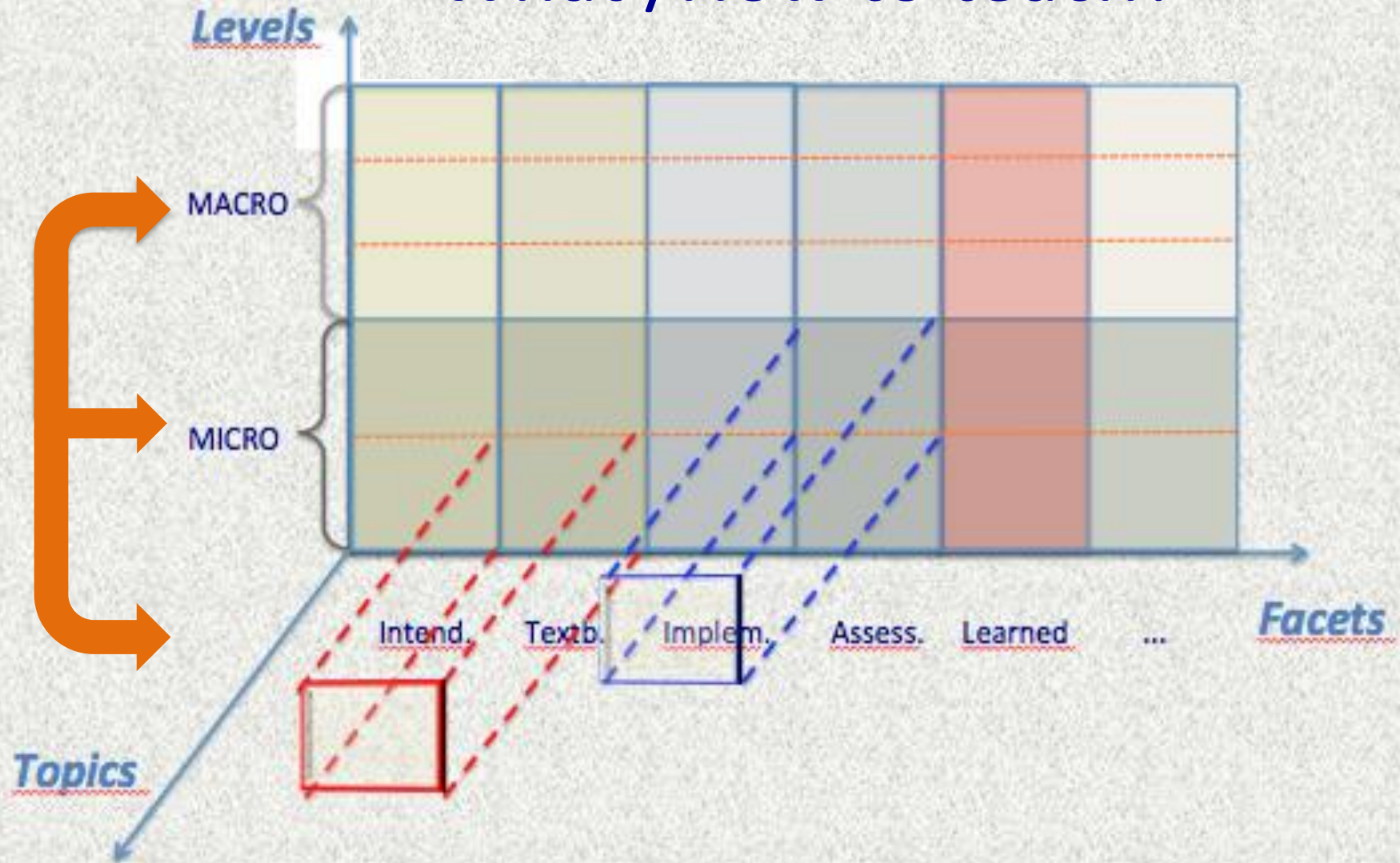
Curricola

Topics

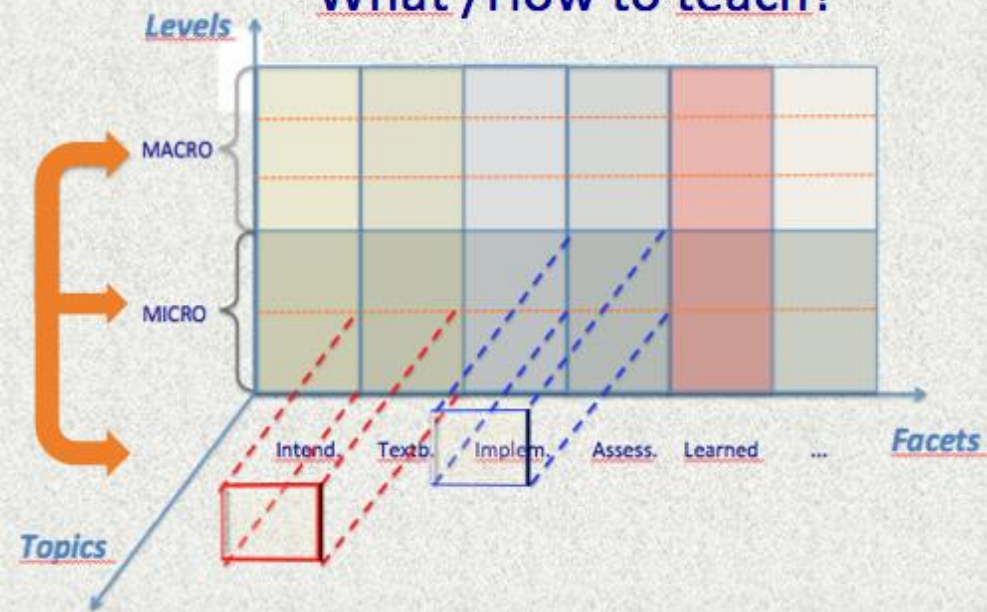
Facets

1a. Designing Curricula: a complex task

What /How to teach?



What /How to teach?



*The development of a school curriculum must take into account both the **instrumental** and the **cultural** function of mathematics: it is an essential instrument for a quantitative understanding of reality and logically coherent and systematic knowledge characterised by a strong cultural unity.*

1b. Comparing Curricula: the risk of ambiguity

The structural description of curricula can be an antidote against possible mistakes done when analysing curricula for different purposes: comparisons, assessments, content, ...

The complexity of the structure shows how complex any analysis should be and how easy can be drawing superficial or wrong conclusions because some level or facet is forgotten.

As E. Silver (2009) points out, the intentions and actions of the consumers of international comparisons often rest on too simplistic assumptions about the relationship among the various interactive constituent parts of the education system in a country: official curriculum goals, textbooks used in schools, teaching practices, teacher preparation and ongoing support, and student learning outcomes.

This superficial approach can have negative consequences: for example policy makers could be induced to undertake programs and give recommendations, which rest on partial or misinterpreted data.

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Assessment

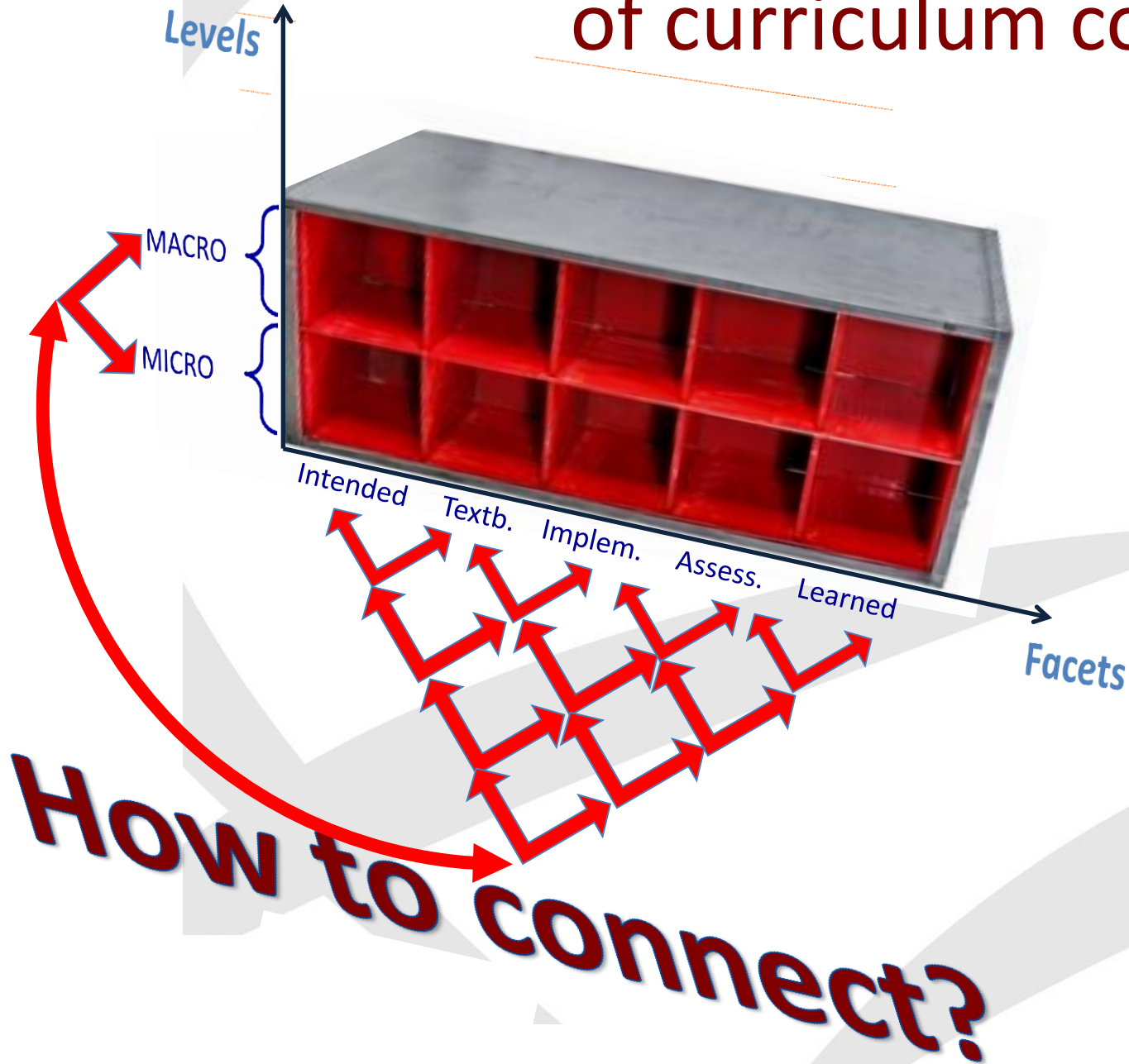
Teachers' education programs

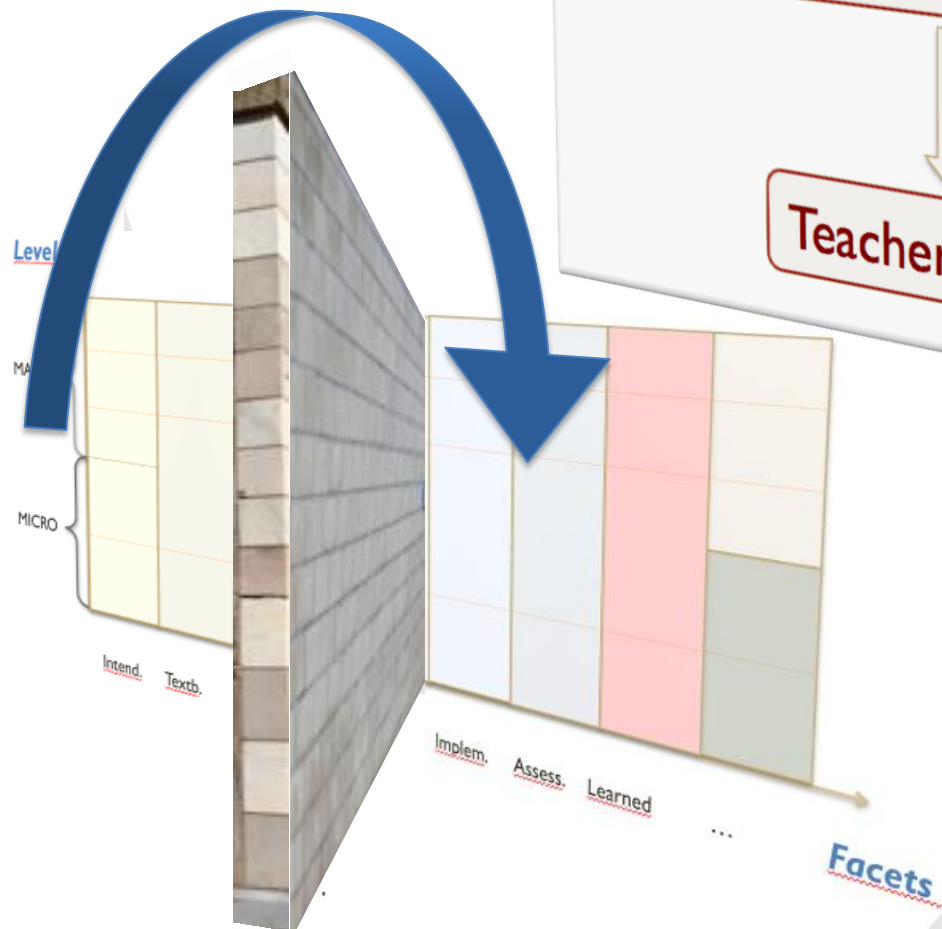
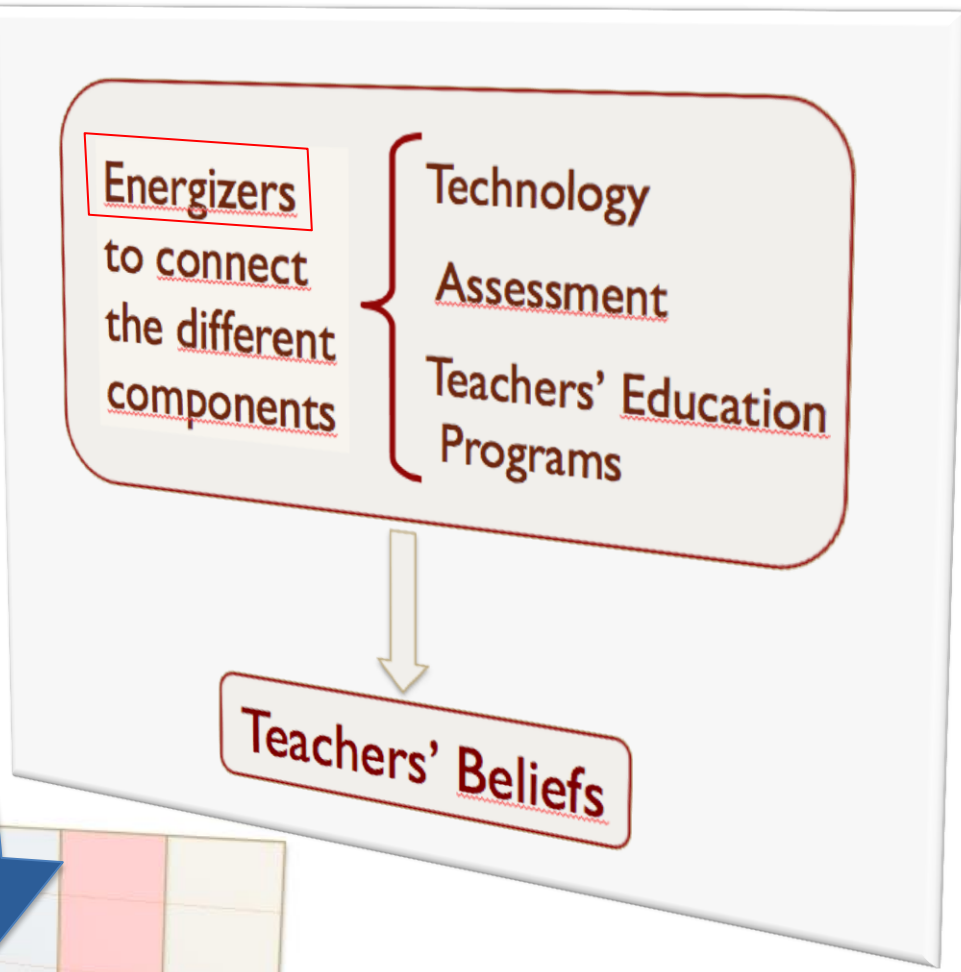
Hindrances for learning:

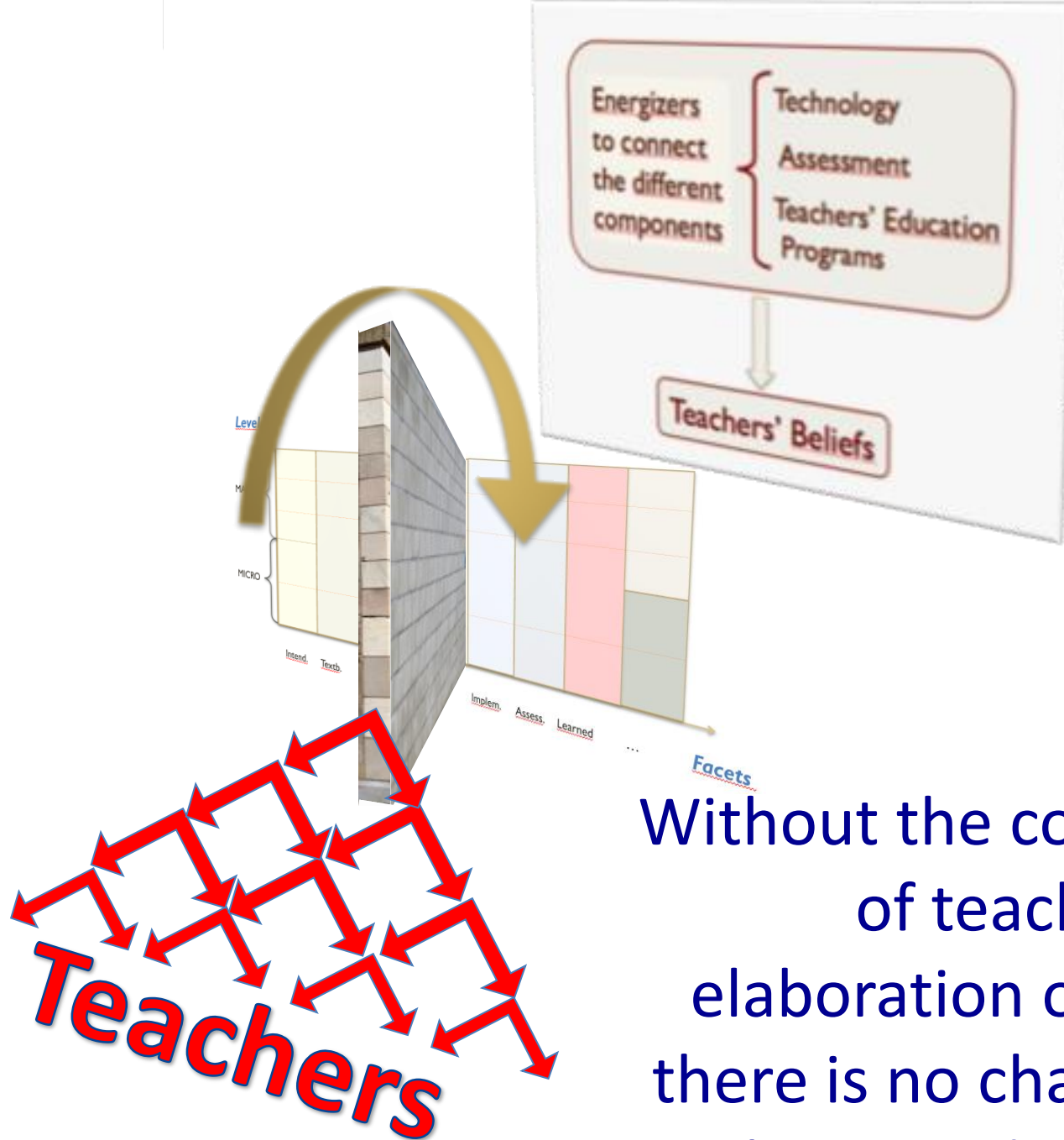
Technology

Gender gap in Mathematics

The compartmentalization of curriculum components







Without the contribution of teachers to the elaboration of curricula there is no change in the implemented curriculum.

A specific frame: Teachers' Beliefs and Curricula

It is well known that there can be a great difference between the adopted curriculum and the intentions of people or institutions, who designed it, and the ways according to which the curriculum materials (e.g. textbooks) are concretely enacted in the classrooms. The influence of teachers' beliefs on this gap between the various facets of curricula is remarkable (e.g. see Kulm & Li, 2009).

Many studies, basing on PISA and TIMSS data, show that, notwithstanding the emphasis given in many curricula to high-level cognitive processes (e. g. reasoning and problem solving), the believes of teachers about the effective ways they can teach mathematics to “mean achievers” students are at the origin of the limited opportunities they give to their students in such processes in their lessons (Silver, 2009).

These beliefs persist in the schools notwithstanding many researches show that a better learning occurs exactly in those classrooms where teaching is based on high-level cognitive demands and not only on stressing procedural instruction.

Research conducted in the past decade or more in a variety of different classroom contexts has found that greater student learning occurs in classrooms where the high-level cognitive demands of mathematical tasks are consistently maintained throughout the instructional episode.

Silver, E. Cross-national comparisons of mathematics curriculum materials: what might we learn? *ZDM Mathematics Education* (2009) 41:827–832.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record*, 110(3), 608–645.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform.

Tarr, J. E., Reys, R. E., Reys, B. J., Chavez, O., Shih, J., & Osterlind, S. J. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education*, 39, 247–280.

Energizers to
connect the
different
components

- 2a. Technology
- 2b. Assessment
- 2c. Teachers' Education Programs

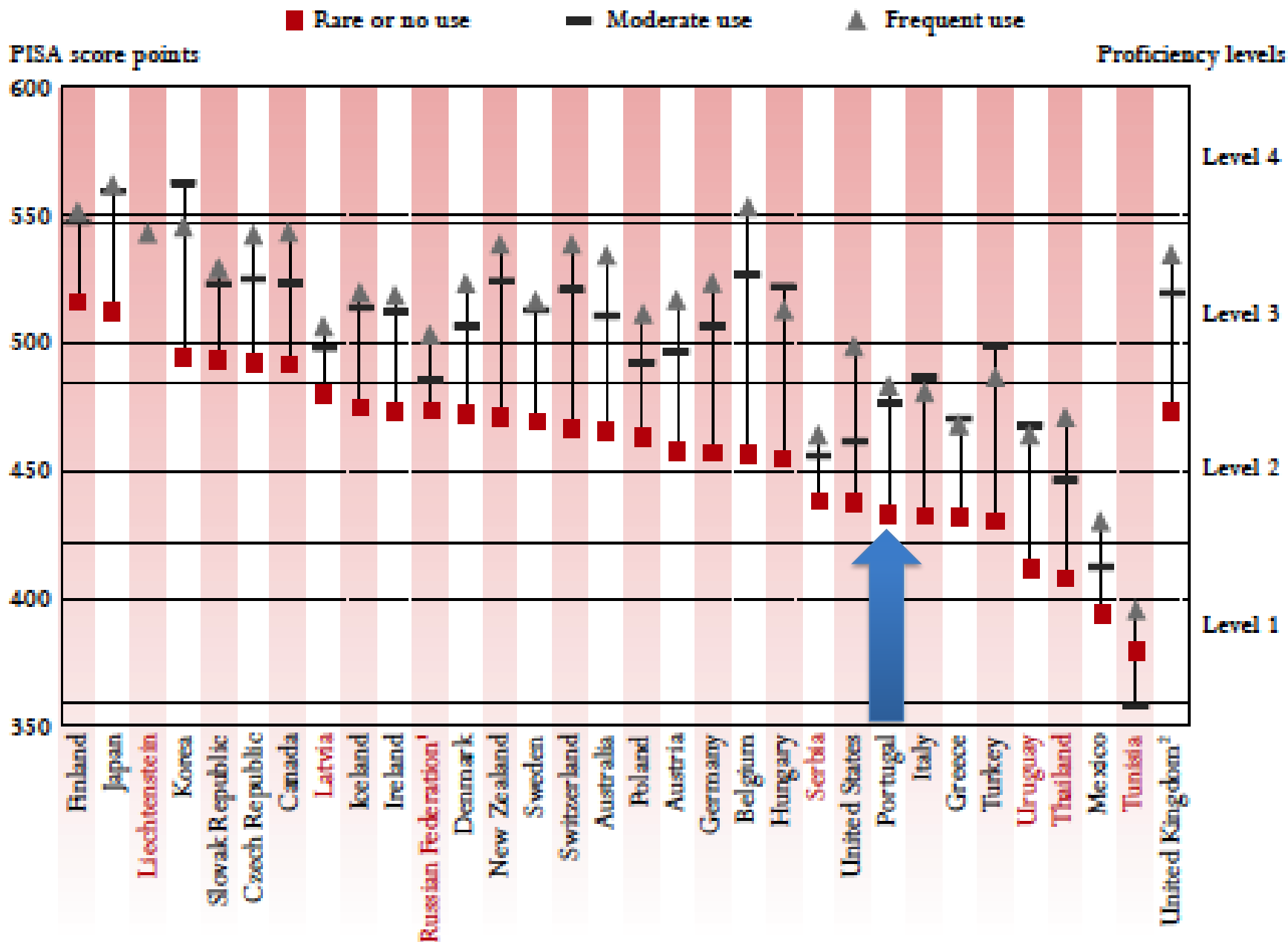


The Technology Principle of NCTM Principles & Standards

Technology:

- *enhances mathematics learning ;*
- *supports effective mathematics teaching ;*
- *influences what mathematics is taught.*

Figure 4.5a ■ Frequency of use of computers at home and student performance in mathematics



Towards the digital school !

?



Energizers to
connect the
different
components

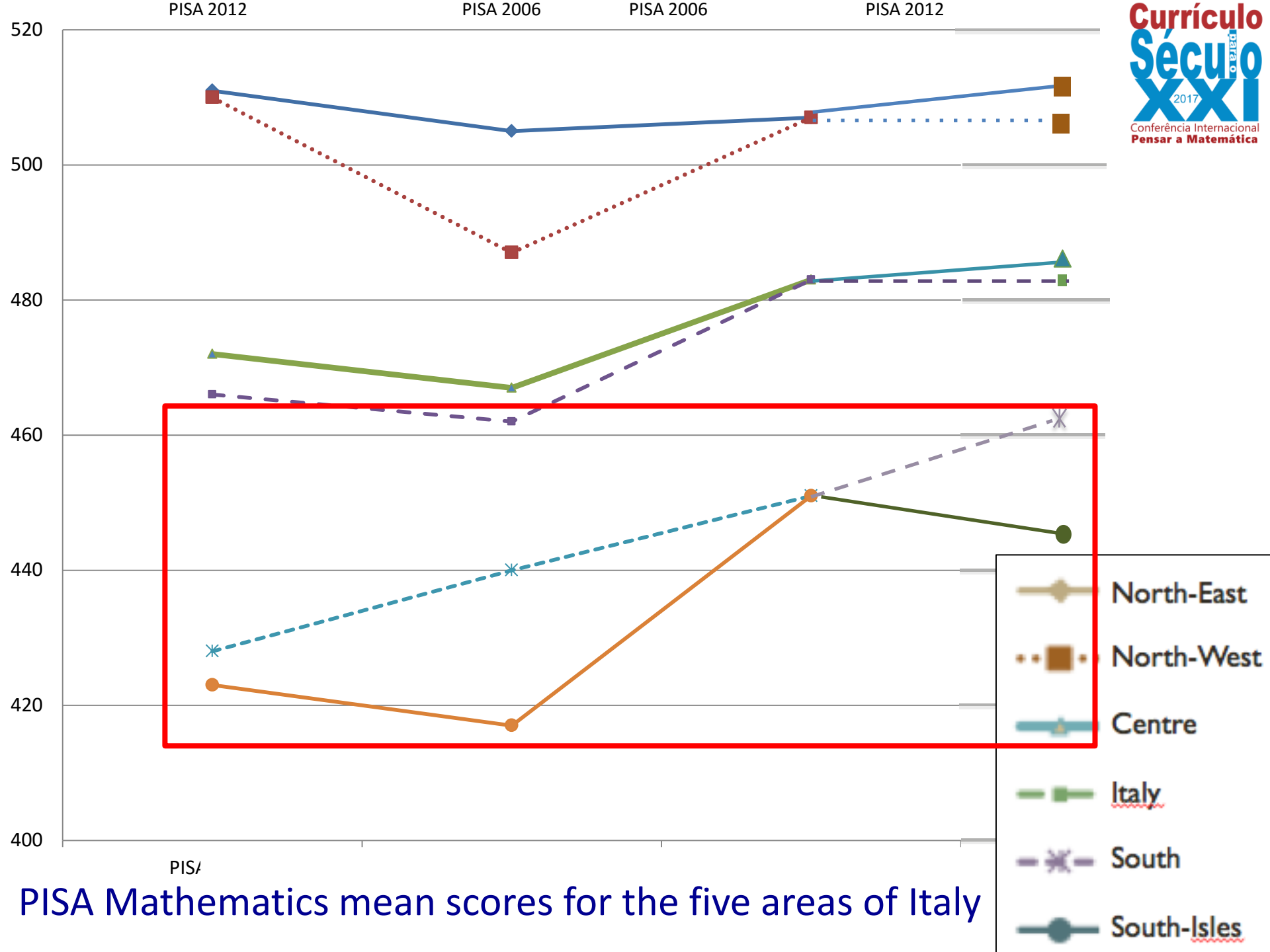
2a. Technology

2b. Assessment

2c. Teachers'
Education Programs

CONS: The risk of an education test-oriented

PROS: Possible positive effects in schools' practices



PISA Mathematics mean scores for the five areas of Italy

Energizers to
connect the
different
components

2a. Technology

2b. Assessment

2c. Teachers'
Education Programs



From the UMI curriculum to the Digital school in Italy

1. 2001-2005: *Mathematics for the Citizen*: an intended curriculum with 200 examples of teaching situations (→ textbook curriculum)
2. 2006-2015: *M@t.abel*: interactive online activities with teachers for improving the implemented curriculum
3. 2012: the official National intended curriculum.
4. 2015: Towards the digital school

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DI
LAVORO



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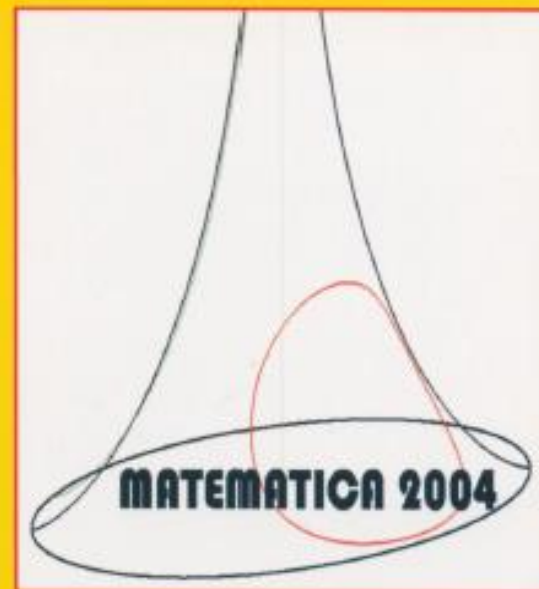
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**Liceo Scientifico
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Lugo di Romagna
(Ravenna)**

Grade 13



**La Matematica
per il cittadino**

**Attività didattiche e prove
di verifica per un nuovo
curricolo di matematica**

**Quinta classe
del ciclo secondario
di secondo grado**

PROJECTS TRAINING FOR TEACHERS

m@t.abel

Mathematics for students on the threshold of the third millennium



Unione Europea

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EUROPEI



2007-2013



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e nazionali per lo sviluppo e la coesione sociale

COMPETENZE PER LO SVILUPPO (FSE)

<http://mediarepository.indire.it/iko/uploads/allegati/M7PWITOE.pdf>

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Paideia 2.0: an example

1



PISA and Technology



OECD PISA

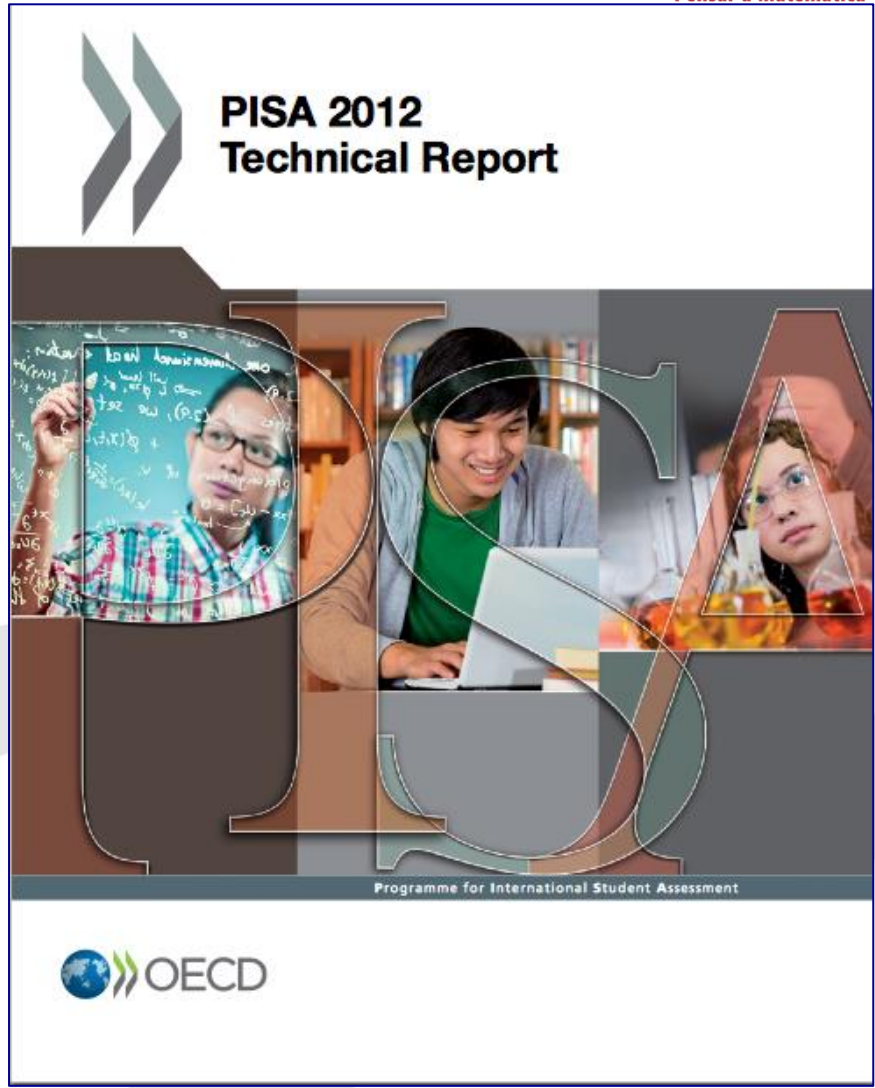


Figure 4.5a ■ Frequency of use of computers at home and student performance in mathematics

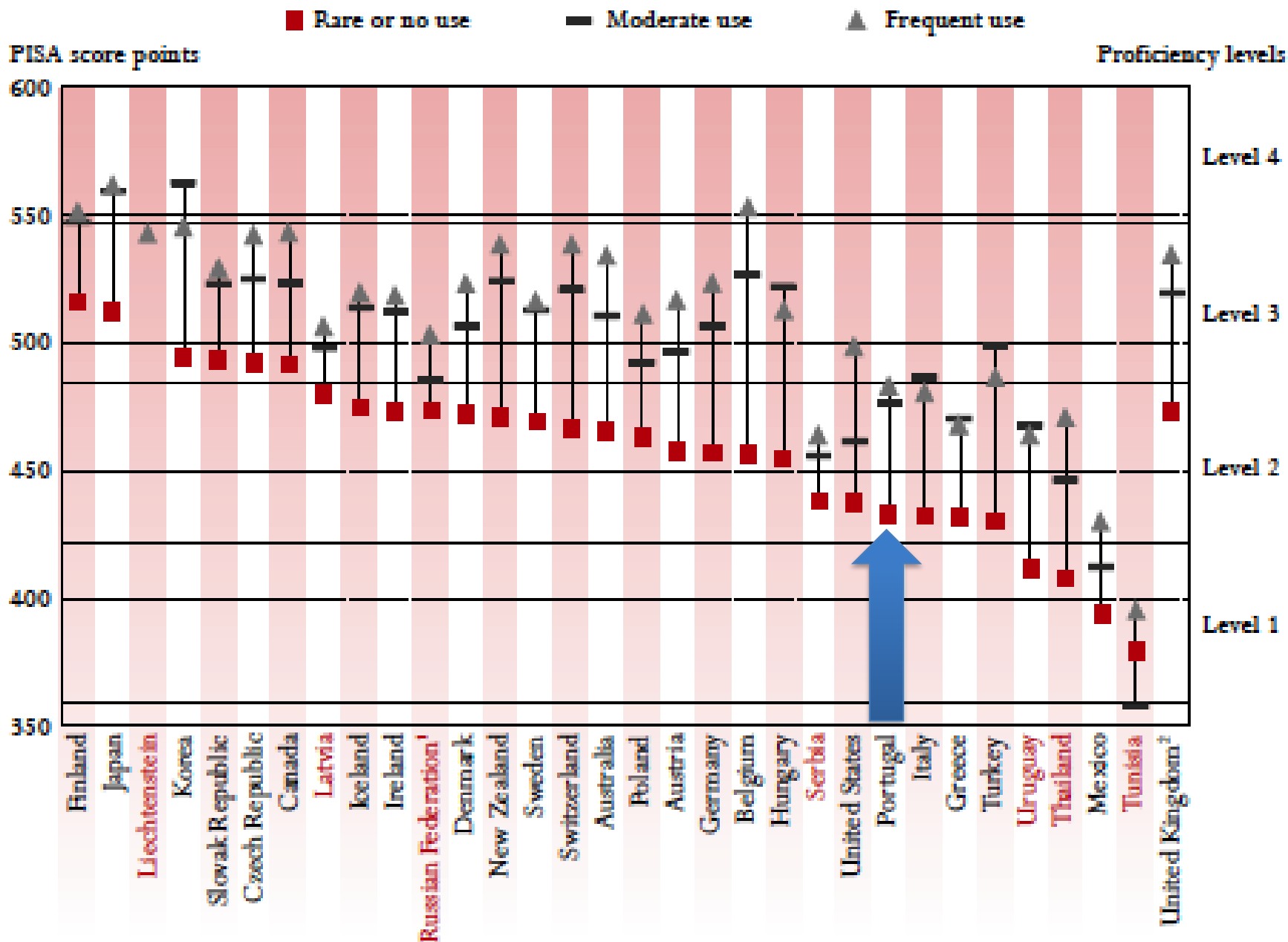


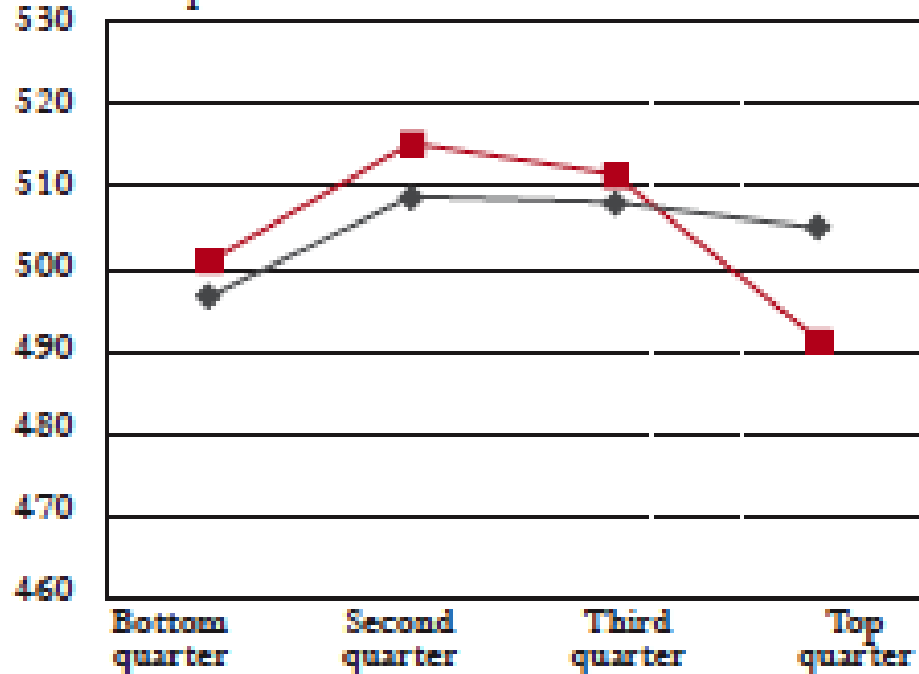
Figure 4.6 ■ Students' use of ICT and OECD average performance in mathematics and reading, by quarter of the indices

■ Index of ICT Internet/entertainment use

◆ Index of ICT program/software use

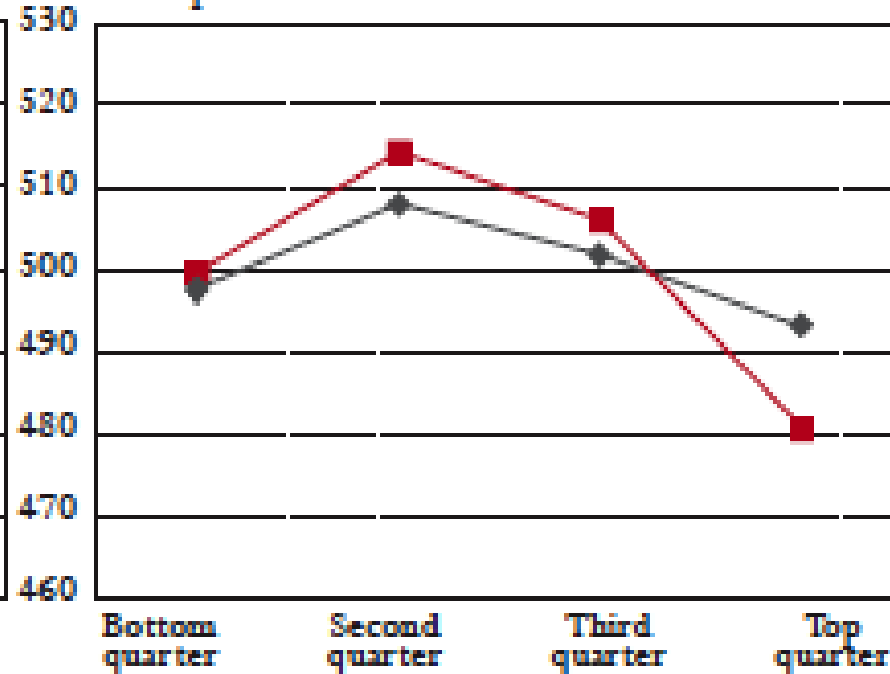
Mathematics performance

PISA score points



Reading performance

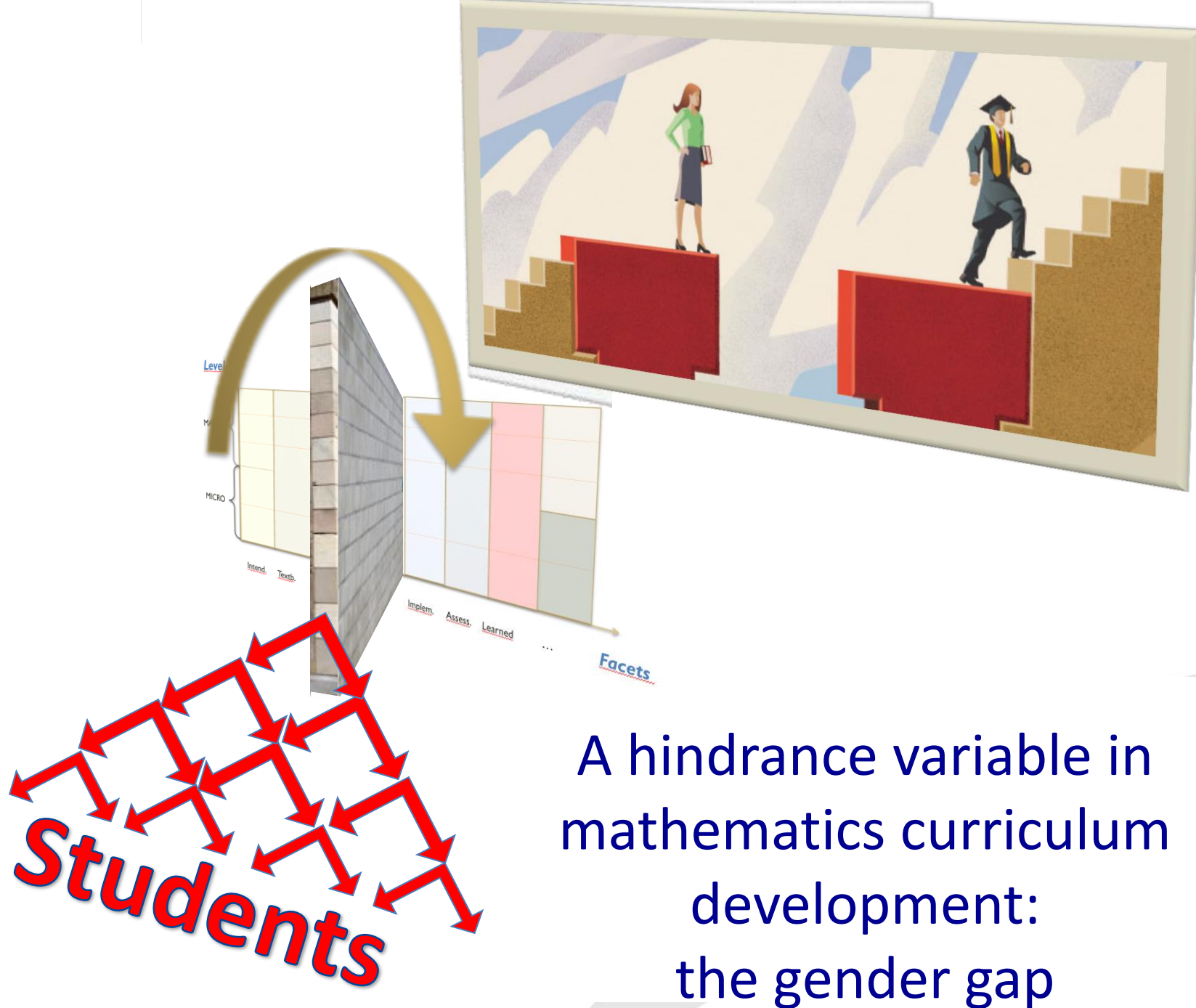
PISA score points



PISA shows that even when most students have easy access to new media, inequalities persist in the way they use these tools.

The use of online media depends on the student's own level of skills, motivation, and support from family, friends and teachers, which vary across socio-economic groups.

Ensuring that every child attains a baseline level of proficiency in reading will do more to create equal opportunities in a digital world than will expanding or subsidising access to high-tech devices and services.



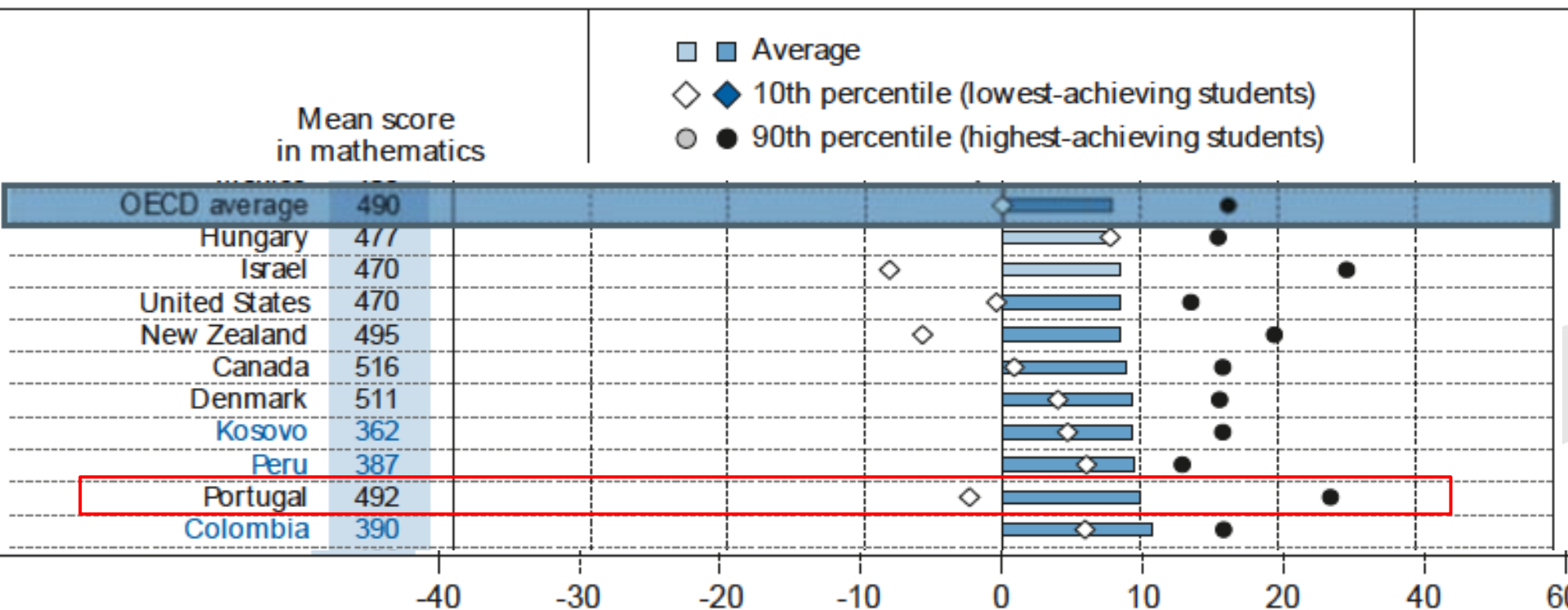
A hindrance variable in mathematics curriculum development: the gender gap

Boys keep doing better than girls in math tests. According to PISA, the average gender differential within OECD countries in mathematics at age 15 is 0.11 standard deviations in favour of males (OECD 2015).

Exhibit 1.10: Average Mathematics Achievement by Gender

Country	Girls		Boys		Difference (Absolute Value)
	Percent of Students	Average Scale Score	Percent of Students	Average Scale Score	
† Hong Kong SAR	46 (1.5)	609 (3.8)	54 (1.5)	619 (2.8)	10 (3.3)
2 Portugal	49 (0.8)	<u>536 (2.4)</u>	51 (0.8)	<u>547 (2.5)</u>	11 (2.2)
Slovak Republic	48 (0.9)	493 (3.0)	52 (0.9)	504 (2.6)	11 (2.6)
2 Spain	49 (0.9)	499 (2.7)	51 (0.9)	511 (2.7)	12 (2.4)
Croatia	49 (0.8)	496 (2.1)	51 (0.8)	508 (2.3)	12 (2.7)
2 Italy	49 (0.7)	497 (2.7)	51 (0.7)	517 (3.0)	20 (2.7)
International Avg.	49 (0.2)	505 (0.5)	51 (0.2)	505 (0.5)	

Figure I.5.10 • Gender differences in mathematics performance
Score-point difference in mathematics (boys minus girls)



The presence of a substantial females' disadvantage in math is of particular importance, because it is likely to be a cause of the critically low share of women choosing STEM disciplines at university, of gender segregation in the labour market, and gender pay gaps (European Commission 2006, 2012, 2015).

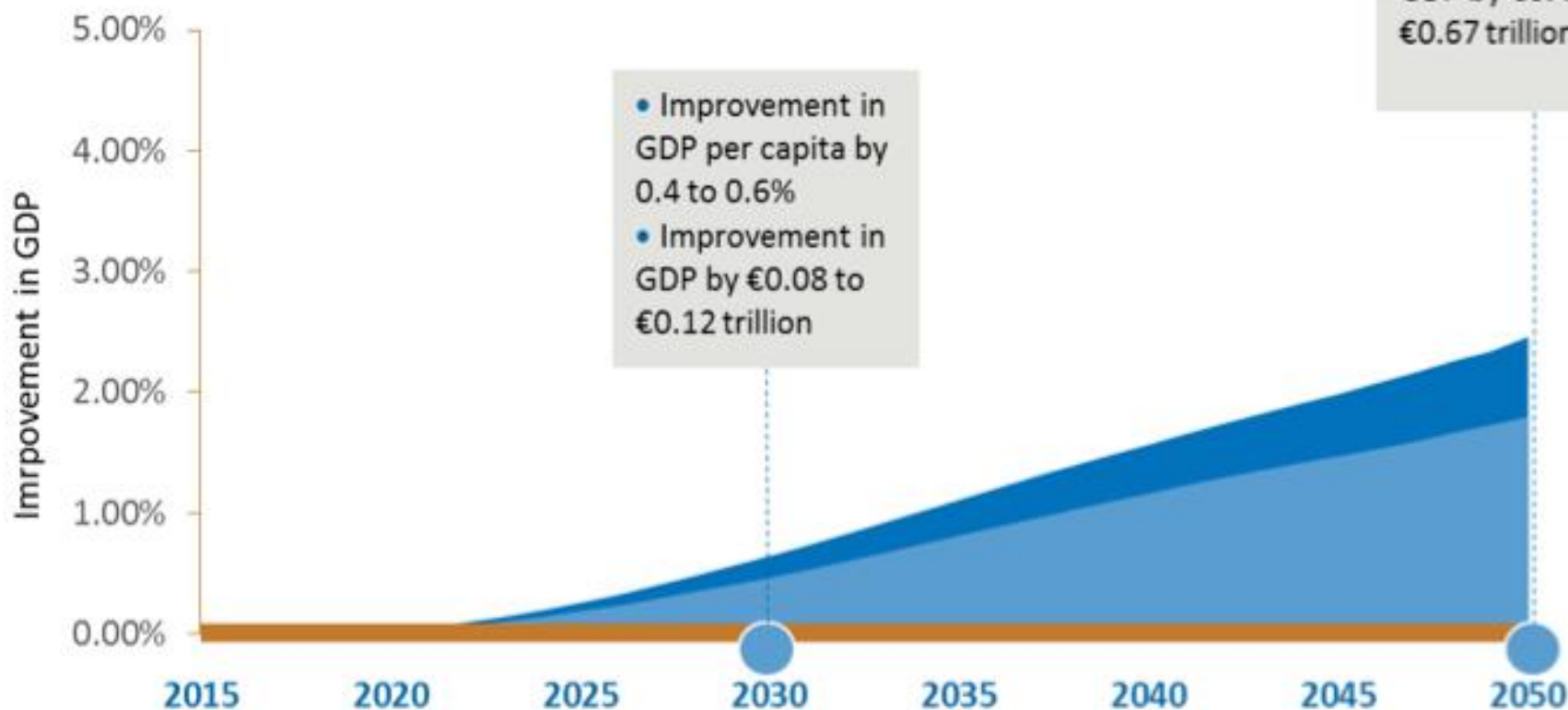
The main finding from researches in USA about primary school (confirmed by PISA surveys at 15 years) is that the math gender gap starts as early as in kindergarten and increases with the age of the child.

Another relevant result is that the math gender gap is higher for top performing students. Initially boys appear to do better than girls among well performers and worse at the bottom of the distribution. By third grade, the gender gap, still larger at the top, appears throughout the distribution.

Survey responses regarding self-concept in maths (year 5 and 6) and on the importance of math for their future life (year 10) show that boys are substantially more confident on their own abilities than girls are, and that they are more aware of the importance in math for their future.

Reducing the gender gap in STEM education and increasing the number of women graduating in STEM subjects leads an increase in labour supply, and in employment. This could help reduce bottlenecks in the labour market.

Effect of closing the educational gap on GDP per capita



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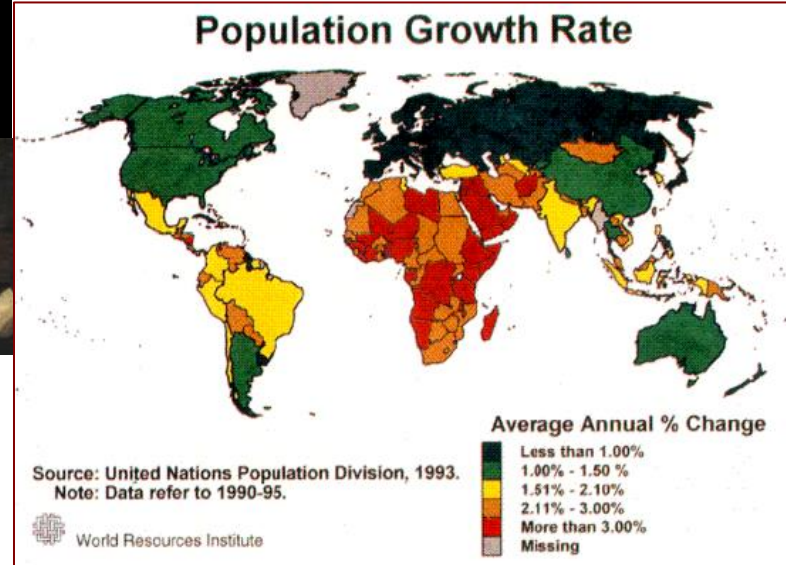
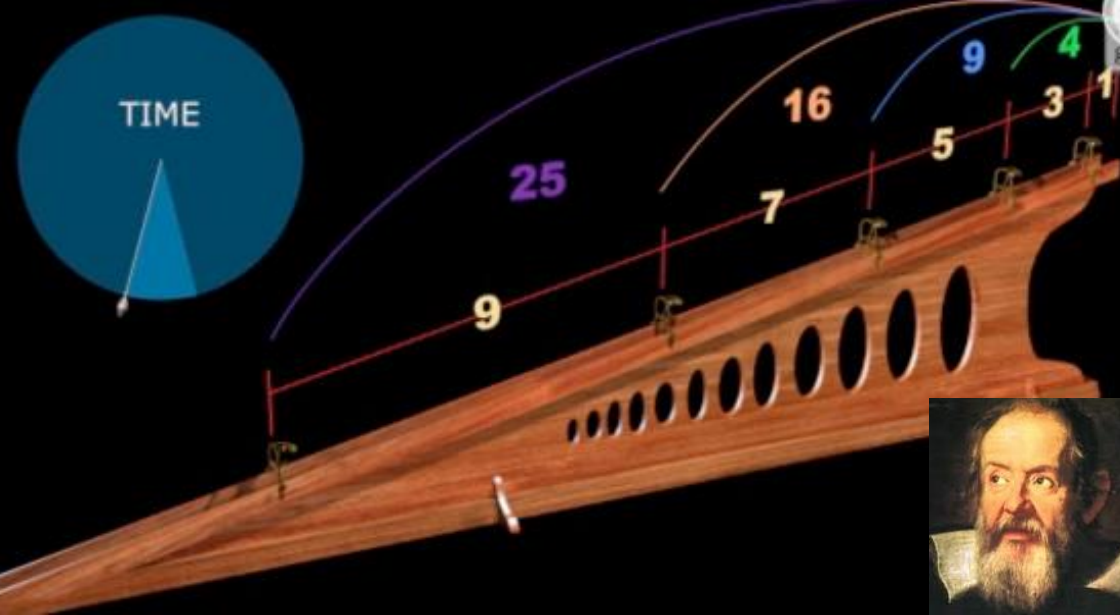
Gender gap in Mathematics

Paideia 2.0: an example

What kind of mathematics
in παιδεία 2.0 ?

An example

Processes of change



Modelling with mathematics

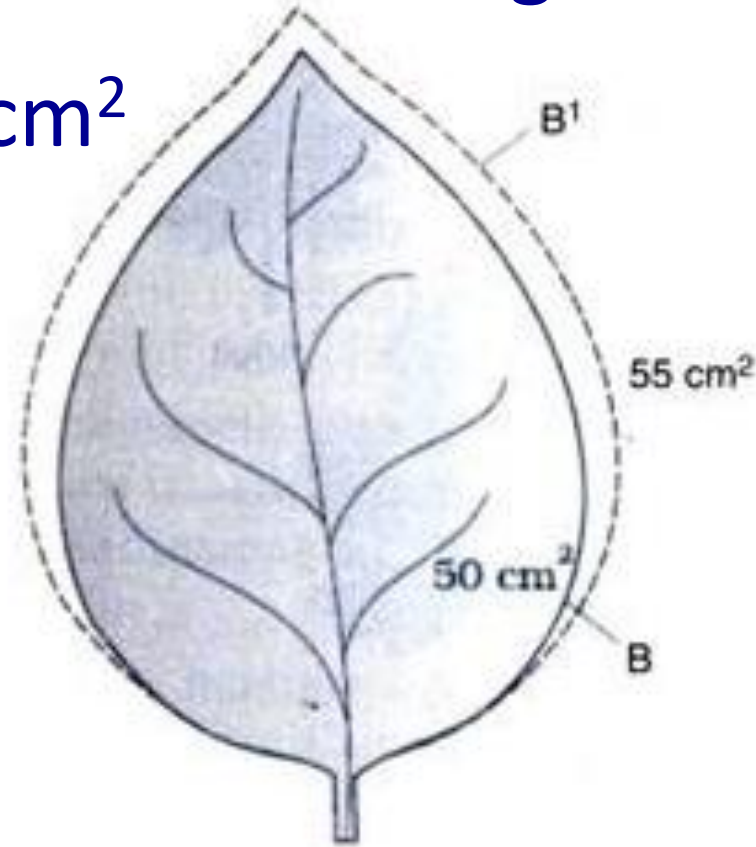
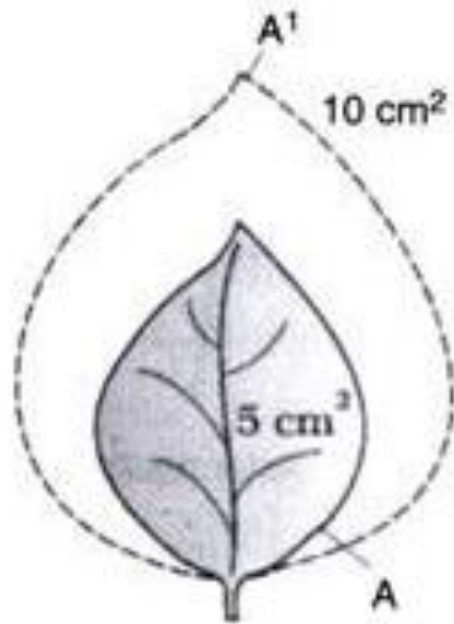


Change is crucial from many standpoints:

- Cognitively: it draws attention;
- Epistemologically: its analysis is the root of the scientific revolution → Calculus
- Culturally: understanding it in climate, economy,... is a crucial issue in XXI cent. society;
- Didactically: Finite differences are a powerful tool, which can easily be implemented with didactical software and allow modelling a variety of phenomena from early grades.

A finer idea of change

$$\Delta A = 5 \text{ cm}^2$$



The relative change $\Delta_r A = \Delta A / A$

$$\Delta_r = 5 \text{ cm}^2 / 5 \text{ cm}^2$$

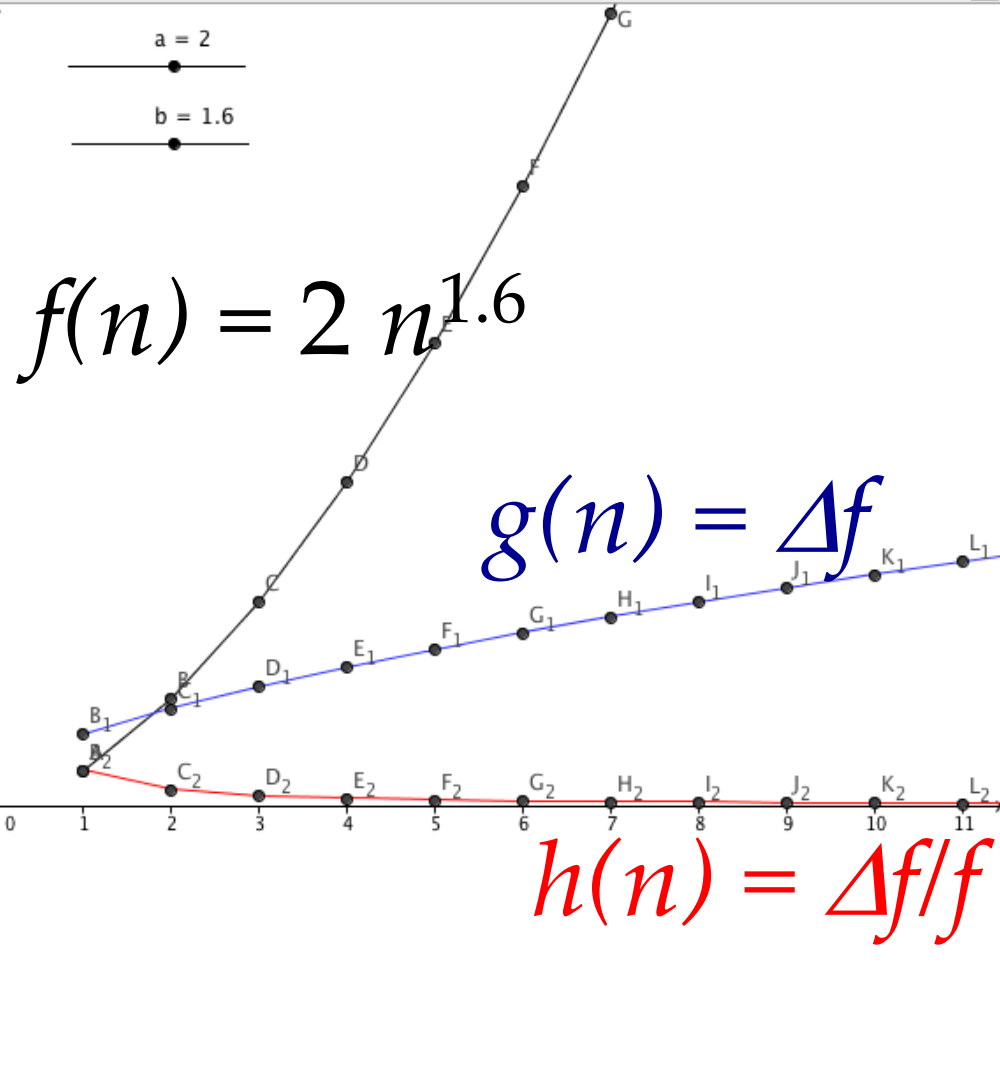
100%

$$\Delta_r = 5 \text{ cm}^2 / 50 \text{ cm}^2$$

10%

Relative differences: polynomes

$$y = a n^b$$

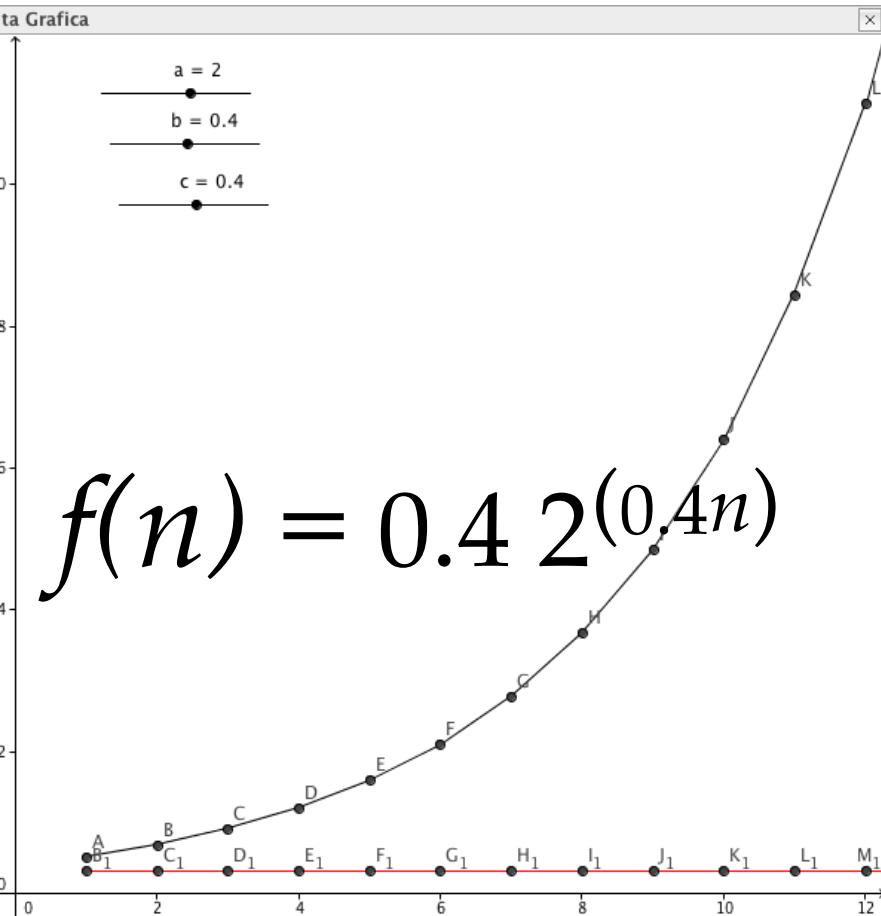


	A	B	C	D	E	F
1	n	a*n^b	n	DELTA B	n	DELTA B / B
2	1	2	1	4.06	1	2.03
3	2	6.06	2	5.54	2	0.91
4	3	11.6	3	6.78	3	0.58
5	4	18.38	4	7.89	4	0.43
6	5	26.27	5	8.9	5	0.34
7	6	35.16	6	9.84	6	0.28
8	7	45	7	10.72	7	0.24
9	8	55.72	8	11.55	8	0.21
10	9	67.27	9	12.35	9	0.18
11	10	79.62	10	13.12	10	0.16
12	11	92.74	11	13.85	11	0.15
13	12	106.59	12	14.56	12	0.14
14	13	121.15	13	15.25	13	0.13
15	14	136.41	14	15.92	14	0.12
16	15	152.33	15	16.57	15	0.11
17	16	168.9	16	17.2	16	0.1
18	17	186.1	17	17.82	17	0.1
19	18	203.92	18	18.43	18	0.09
20	19	222.35	19	19.02	19	0.09
21	20	241.37	20	19.6	20	0.08
22	21	260.96	21	20.17	21	0.08
23	22	281.13	22	20.72	22	0.07
24	23	301.85	23	21.25	23	0.07
25	24	323.12	24	21.76	24	0.06
26	25	344.93	25	22.25	25	0.06



Relative differences: exponentials

$$y = c a^{(bn)}$$



Vista Foglio di calcolo

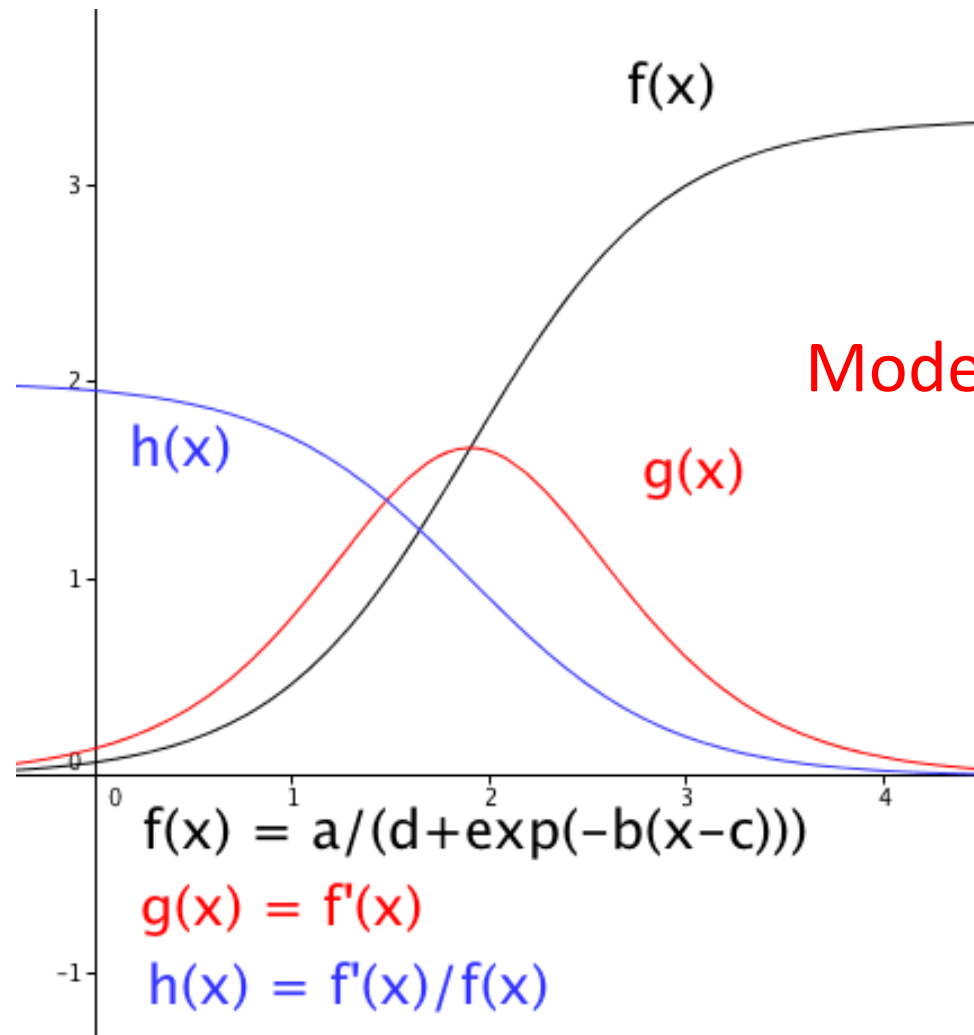
	A	B	C	D	E	F	G	H
1	n	a^n	a^(bn)	n	c*a^(b*n)	DELTA E	n	DELTA E/E
2	1	2	1.32	1	0.53	0.17	1	0.32
3	2	4	1.74	2	0.7	0.22	2	0.32
4	3	8	2.3	3	0.92	0.29	3	0.32
5	4	16	3.03	4	1.21	0.39	4	0.32
6	5	32	4	5	1.6	0.51	5	0.32
7	6	64	5.28	6	2.11	0.67	6	0.32
8	7	128	6.96	7	2.79	0.89	7	0.32
9	8	256	9.19	8	3.68	1.17	8	0.32
10	9	512	12.13	9	4.85	1.55	9	0.32
11	10	1024	16	10	6.4	2.04	10	0.32
12	11	2048	21.11	11	8.44	2.7	11	0.32
13	12	4096	27.86	12	11.14	3.56	12	0.32
14	13	8192	36.76	13	14.7	4.7	13	0.32
15	14	16384	48.5	14	19.4	6.2	14	0.32
16	15	32768	64	15	25.6	8.18	15	0.32
17	16	65536	84.45	16	33.78	10.79	16	0.32
18	17	131072	111.43	17	44.57	14.24	17	0.32
19	18	262144	147.03	18	58.81	18.79	18	0.32
20	19	524288	194.01	19	77.6	24.8	19	0.32
21	20	1048576	256	20	102.4	32.72	20	0.32
22	21	2097152	337.79	21	135.12	43.17	21	0.32
23	22	4194304	445.72	22	178.29	56.96	22	0.32
24	23	8388608	588.13	23	235.25	75.17	23	0.32
25	24	16777216	776.05	24	310.42	99.18	24	0.32
26	25	33554432	1024	25	409.6			

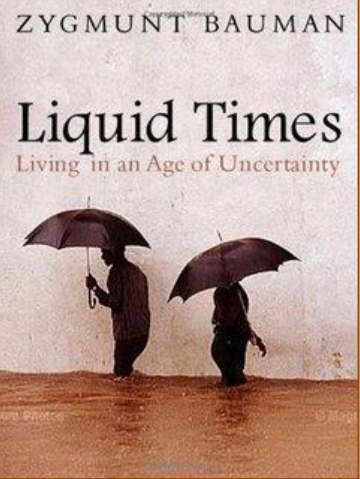
$$h(n) = \Delta f / f$$



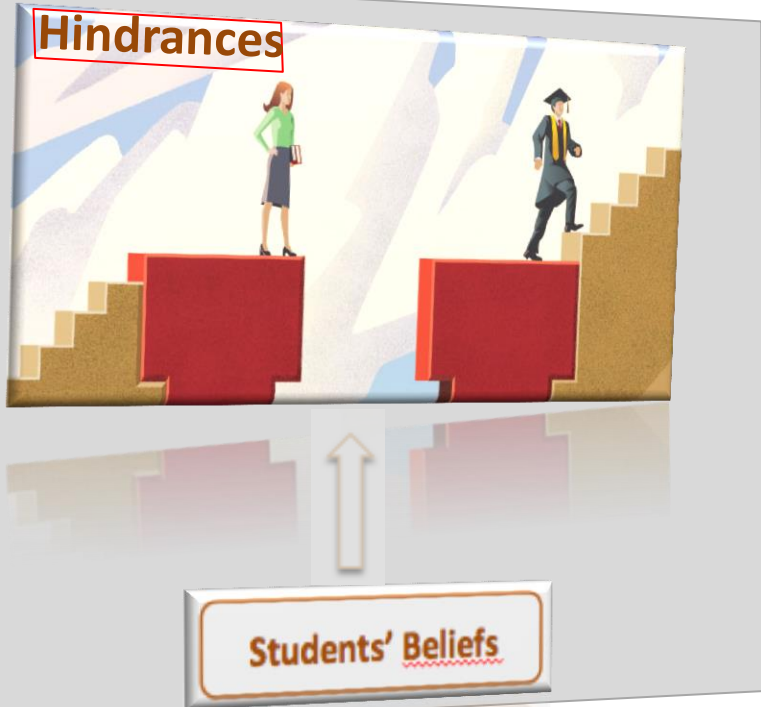
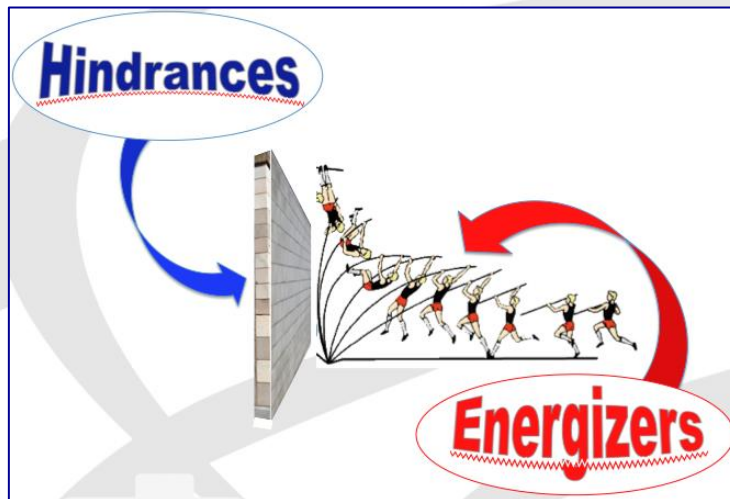
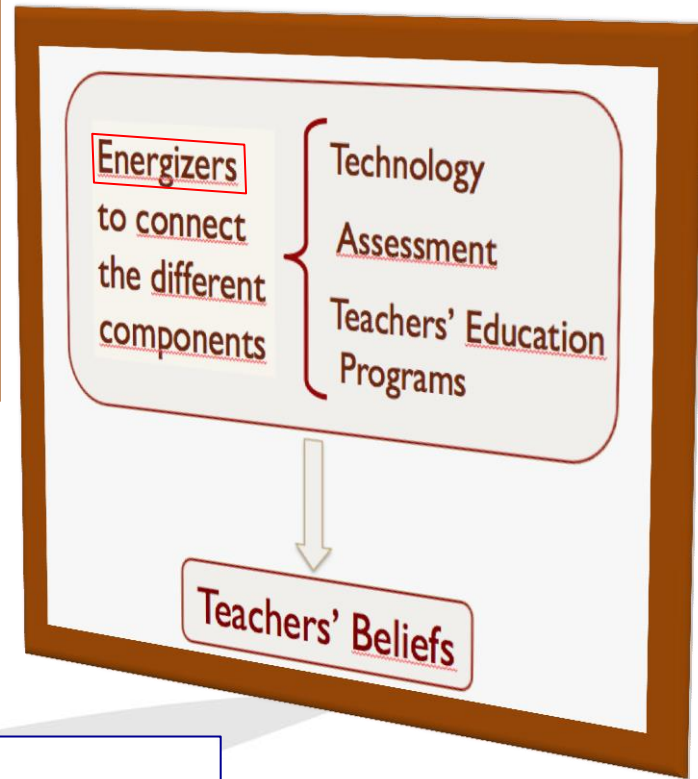
Growth phenomena in Biology and Economy:

reasoning about changes
as education to rational decisions





Currículo
Século para o
XXI
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Pensar a Matemática



Hindrances



Students' Beliefs

ZYGMUNT BAUMAN

Liquid Times

Living in an Age of Uncertainty



**Currículo
 Século
 XXI**
 2017
 Conferência Internacional
 Pensar a Matemática

Energizers
 to connect
 the different
 components

Technology
 Assessment
 Teachers' Education
 Programs

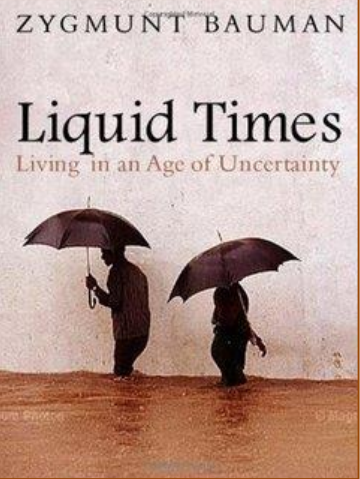
Teachers' Beliefs

Hindrances



The moral of my talk:

To break hindrances and transform them into energizers we need a critical attention to the ongoing changes in society and a careful action on teachers and students beliefs. Otherwise also the best intended curriculum will produce little change in the implemented and learned ones.



Hindrances



Energizers
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Technology
Assessment
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Obrigado

Students' Beliefs

Teachers' Beliefs

**Curriculo
Sociedade
XXI**
2017
Conferência Internacional
Pensar a Matemática

Hindrances



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